Math 32 – Workshop #14

1. Find the limits. Note which functions are continuous at the limit point and which functions have a removable discontinuity.

   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{3x^2-y^2+5}{x^2+y^2+2}$

   (b) $\lim\limits_{(x,y)\to(1,1)} \dfrac{x^2-2xy+y^2}{x-y}$

   (c) $\lim\limits_{(x,y)\to(1,1)} \dfrac{xy-y-2x+2}{x-1}$

   (d) $\lim\limits_{(x,y)\to(\frac{\pi}{2},0)} \dfrac{\cos y+1}{y-\sin x}$

   (e) $\lim\limits_{(x,y)\to(1,1)} \ln|1+x^2y^2|$

   (f) $\lim\limits_{(x,y)\to(2,2)} \dfrac{x+y-4}{\sqrt{x+y}-2}$

   (g) $\lim\limits_{(x,y)\to(2,-3)} \left(\dfrac{1}{x}+\dfrac{1}{y}\right)^2$

   (h) $\lim\limits_{(x,y)\to(4,3)} \dfrac{\sqrt{x}-\sqrt{y+1}}{x-y-1}$

   (i) $\lim\limits_{(x,y)\to(0,0)} \dfrac{e^y\sin x}{x}$

   (j) $\lim\limits_{(x,y)\to(2,0)} \dfrac{\sqrt{2x-y}-2}{2x-y-4}$

2. Prove that the limit does not exist.

   (a) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4}{x^4+y^2}$

   (b) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y}{y}$

   (c) $\lim\limits_{(x,y)\to(0,0)} \dfrac{xy^4}{x^3+y^6}$

   (d) $\lim\limits_{(x,y,z)\to(0,0,0)} \dfrac{yz}{x^2+4y^2+9z^2}$

3. The wind-chill index W is the perceived temperature when the actual temperature is T and the wind speed is v, so we can write $W=f(T,v)$. The table below represents some values of this function.

T \ v	30	40	50	60	70
-15	-26	-27	-29	-30	-30
-20	-33	-34	-35	-36	-37
-25	-39	-41	-42	-43	-44

   (a) What row or column would we use to compute $f(-20,v)$? $f(T,60)$? Explain what $f(-20,v)$ and $f(T,60)$ would represent.

   (b) If we were to compute $\dfrac{\partial W}{\partial T}$ and $\dfrac{\partial W}{\partial v}$, explain what they would represent.

4. Find the first partial derivatives of the function. Write your answer with no negative exponents or complex fractions.

   (a) $f(x,y)=x^4y^3+8x^2y-x+4y$

   (b) $f(x,y)=x^2\tan(xy)$

   (c) $f(s,t)=\dfrac{3st^2}{5s^4+2t}$

   (d) $f(u,v)=\ln\left(u^2-\sqrt{v}\right)$

   (e) $f(x,y)=e^{\frac{2x^3}{(y+1)}}$

   (f) $f(x,y)=y^x$