Math 32 - Workshop #18-2 for §14.7 extreme values

Second Derivatives Test. Suppose the second partial derivatives of f are continuous on a disk with a center (a,b) and suppose that $f_x(a,b) = f_y(a,b) = 0$. Define $D(a,b)$, where $D(a,b) = f_{xx}(a,b)f_{yy}(a,b) - f_{xy}(a,b)$. For the functions below, find the candidates (a,b), compute $D(a,b)$ and then find any values of the local minimum(s), local maximum(s), saddle point(s), or where the test fails (be sure to label which is which). Try using a program such as 3D Desmos or Geogebra 3D calculator to sketch a graph of the function and verify your answers.

1. $f(x,y) = -x^3 + 4xy - 2y^2 + 1$

2. $f(x,y) = x^2 y^2$

The Extreme Value Theorem tells us that a continuous function f on a closed, bounded set in $\mathbf{R}^2$ attains a maximum and a minimum value. For the problem below, first sketch a graph of the region R and then find the candidates for the absolute extrema in the interior and the candidates for the absolute extrema on the boundary. Once you have found all possible candidates, test to find the absolute extrema and where they occur.

3. $f(x,y) = x^2 - 4xy + 5$, $R = \{(x,y) \mid 1 \le x \le 4, \ 0 \le y \le 2\}$