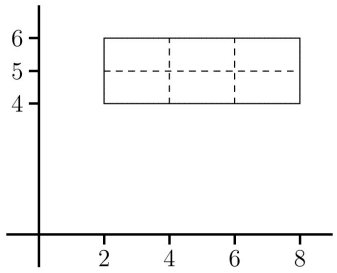
Math 32 – Workshop #19

1. If we want to integrate over a rectangle $R = \{(x,y)\big|a \le x \le b, c \le y \le d\} = [a,b] \times [c,d]$, then we have the definition $\iint\limits_R f(x,y)dA = \lim\limits_{m,n\to\infty} \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A_{ij}$. To estimate this integral, we can use $\iint\limits_R f(x,y)dA \approx \sum\limits_{i=1}^{m}\sum\limits_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A_{ij}$

   We want to use this to estimate the volume of the solid that lies below the surface $z = 4xy$ and above the rectangle $R = [2,8] \times [4,6]$ by using a Riemann sum and the 3-by-2 grid below (then $m = 3$ and $n = 2$). Choose the points to evaluate the heights to be the lower left corner.

   $(x_{11}^*, y_{11}^*) =$
   $(x_{12}^*, y_{12}^*) =$
   $(x_{21}^*, y_{21}^*) =$
   $(x_{22}^*, y_{22}^*) =$
   $(x_{31}^*, y_{31}^*) =$
   $(x_{32}^*, y_{32}^*) =$



   Using the appropriate definitions and proper notation, estimate this volume.

2. Integrate completely.

   (a) $\int_0^{\pi/2}\int_0^{\pi} (xy + \sin x)dxdy$

   (b) $\int_0^{2}\int_0^{1} e^{x+y}dxdy$

   (c) $\iint\limits_R xe^{x+y}dA$, where $R = [0,2] \times [0,3]$

   (d) $\iint\limits_R \dfrac{xy}{1+x}dA$, where $R = [0,1] \times [0,4]$