Math 32 – Workshop #26

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1. Sketch the region bounded by the paraboloids $z = x^2 + y^2$ and $z = 36 - 3x^2 - 3y^2$. Use cylindrical coordinates to find the volume of this region.

2. Evaluate by first changing to cylindrical coordinates. (*Sketch the region of integration!*)

$$\int_0^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_0^{16-x^2-y^2} (x^2 + y^2)\, dz dy dx$$

3. Sketch the solid whose volume is given by the integral and evaluate.

$$\int_0^{\pi} \int_{\frac{\pi}{2}}^{\pi} \int_1^2 \rho^2 \sin\phi \, d\rho d\phi d\theta$$

4. Evaluate $\iiint_E (x^2 + y^2 + z^2) dV$ using spherical coordinates, where E is the region between the two spheres $\rho = 2$ and $\rho = 4$ and above the cone $\phi = \dfrac{\pi}{3}$ in the first quadrant.

5. Using spherical coordinates, find the volume of the part of the sphere of radius 9 which is inside the sphere, between the cones $\phi = \dfrac{\pi}{3}$ and $\phi = \dfrac{\pi}{6}$, and behind the yz-plane.