Math 32 – Workshop #29

---

1. Evaluate the line integral $\int_C y^2\,dx$, where C is the line connecting the points $(-3,-1)$ and $(5,9)$.

2. Evaluate the line integral $\int_C z\,dx + \sin x\,dy + 3x^2\,dz$, where $C : x = t,\ y = t^2,\ z = e^t,\ 0 \le t \le 1$.

3. Recall that arc length in the plane is given by

$$s(t_0) = \int_a^{t_0} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt,$$

and from this we have that

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2},$$

using t as the parameter. Using differentials, we can write this as

$$ds = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt,$$

and we can integrate a function $f(x,y)$ above a curve C in the plane using

$$\int_a^b f(x,y)\,ds = \int_a^b f(x,y)\sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt.$$

This can be extended to higher dimensions.

Use this to compute $\int_C ye^z\,ds$, where C is the circular helix
$x = \cos t,\ y = \sin t,\ z = t,\ 0 \le t \le 2\pi$.