Stat 1 Pal Worksheet 10: Multiplication Rule for Independent and Non-Independent Events

Name________________________________

In the previous worksheet you learned that:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad (\text{I})$$

And since conditional probabilities are, in general, non-commutative:

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \qquad (\text{II})$$

Solving for $P(A \cap B)$ in equation (I) above you obtain: $P(A \cap B) = P(B)P(A|B)$

Solving for $P(A \cap B)$ in equation (II) above you obtain: $P(A \cap B) = P(A)P(B|A)$

This produces the **Multiplication Rule**:

$$P(A \cap B) = \begin{cases} P(A)P(B|A) \\ \quad or \\ P(B)P(A|B) \end{cases} \qquad (\text{III})$$

The multiplication rule is useful to obtain probabilities of sampling without replacement:

1. Suppose you select at random two students from a class consisting of 12 males and 18 females (a class with a total of 30 students).

a. What is the probability that both students are female?

Hint: So, if F_1 = "the event that the first selected student is a female," and F_2 = "the event that the second selected student is a female," then the probability that both selected students are females can be seen as $P(F_1 \cap F_2)$.

b. What is the probability that both students are males?

c. What is the probability that the first student is a male and the second student is a female?

d. What is the probability that one student is a male and another is a female? (Note that this probability is different from that you obtained in part c above).

2. Suppose that instead of two student you students, three students are randomly selected from the class described in part 1 above. Generalize the multiplication rule to obtain

a. The probability that all three students are female.

b. The probability that all three students are female.

c. The probability that the first and third selected students are female and the second is a male.

Remember if *A* and *B* are independent events, then the prior probability of *B*, P(*B*), is equal to the posterior probability of *B* given *A*, P(*B*|*A*). That is, P(*B*) = P(*B*|*A*). Convince yourself that the this means that the top part of the right-hand-side of equation (III) becomes P(*A*)P(*B*) when substituting P(*B*|*A*) with P(*B*). And vice versa with prior probability of *A*.

Thus, if *A* and *B* are independent the multiplication rule has only one formula:

$$P(A \cap B) = P(A)P(B)$$

Note the use of this last formula in part 3 below:

3. Consider the experiment of tossing a fair coin twice.

a. What is the probability that both tosses come up tails?

b. What is the probability that the first toss comes up tails and the second heads?

c. What is the probability that one toss comes up heads and the other tails?

d. If this experiment consisted on tossing a fair coin 10 times instead of twice. What is the probability that a tail comes up in all 10 tosses?

e. If this experiment consisted on tossing a fair coin 10 times instead of twice. What is the probability that ***at least*** one head comes up in the 10 tosses?