Math 1: Mathematical Reasoning

California State University, Sacramento · Department of Mathematics & Statistics

This is a one-semester course, which satisfies the quantitative reasoning requirement for GE (area B4). It is recommended for students whose majors do not include a specific mathematics requirement. The objectives of the course are the following:

- Show the essence of mathematics - rather than teaching specific techniques in arithmetic, algebra or other areas.
- Help students see some of the quality, elegance, and beauty in mathematics, and overcome any fear of mathematics.
- Enhance precision in the evaluation and expression of ideas, and thereby develop a student's quantitative reasoning skills.

The primary purpose of the course is to give students an understanding of some of the vocabulary, methods and reasoning of mathematics. The focus is on the ideas of mathematics and on giving students an understanding of why results hold - and not on learning specific results, techniques, or skills. Students will be given periodic writing assignments that encourage them to think through concepts of the course.

Catalog Description

Recommended for students whose majors do not include a specific mathematics requirement. Objectives are to show some of the essence and quality of mathematics, and to enhance precision in the evaluation and expression of ideas, thereby developing a student's quantitative reasoning skills. Designed to give students an understanding of some of the vocabulary, methods, and reasoning of mathematics with a focus on ideas. **Graded**: Graded Student. **Units**: 3.0.

Prerequisites

None.

Area B-4 Mathematical Concepts and Quantitative Reasoning Student Learning Outcomes

Students will be able to:

1. Solve problems by thinking logically, making conjectures, and constructing valid mathematical arguments.
2. Make valid inferences from numerical, graphical and symbolic information.
3. Apply mathematical reasoning to both abstract and applied problems, and to both scientific and non-scientific problems.

Text

To be determined in consultation with the instructor and the Math 1 Coordinator. Texts will be recommended for use in the course, and a text will be chosen for any sections that are not taught by full-time faculty. (The recommendations of texts are intended to indicate a reasonable level for the course.) A typical text for classes not taught by full-time faculty would be *Excursions in Modern Mathematics* by Peter Tannenbaum.

Coverage

Coverage will be determined from the list of topics below, and the chosen text or classroom materials, in a manner consistent with course objectives. Handouts will be given outlining the material to be covered as the course progresses.

Writing Component

This is an area B4 GE course and has a writing component. To satisfy the writing requirement graded assignments involving writing and understanding of complex technical prose, interpretation of theoretical ideas, and the use of mathematical ideas will be part of the course.

Topics

Course content will normally be selected from the topics listed below. These topics can be applied to a wide variety of subject areas, and the selection of topics will be made by the instructor in order to best satisfy the purposes of the course (in general, the instructor should consider three topics as being sufficient for the course).

I. Election/Voting Theory
   - A. The Basic Elements of an Election
   - B. Methods for Determining a Winner (Plurality, Borda Count, Plurality-with-Elimination, Pairwise Comparison)
   - C. Fairness Criteria and Arrow's Impossibility Theorem

II. The Mathematics of Power
   - A. Overview of Weighted Voting
   - B. Banzhaf Power Index
   - C. Shapley-Shubik Power Index

III. Fair-Division (The Mathematics of Sharing)
   - A. Introduction to Fair-Division
   - B. Methods (Divider-Chooser, Lone-Divider, Sealed Bids, Markers)

IV. Apportionment
   - A. Introduction to Apportionment
   - B. Problems and Methods (incl. selections from Hamilton's, Jefferson's, Adams and Webster's, and the Huntington-Hill Method)
   - C. Quota Rule and Paradoxes

V. Graph Theory (option 1)
   - A. Introduction to Graph Theory
   - B. Euler's Theorem
   - C. Eulerizing a Graph
   - D. Applications/Puzzles of Graph Theory (Tracing Pictures, Bridges of Konigsberg, Rooms and Walls, Map Coloring)

VI. Graph Theory (option 2)
   - A. Introduction to Graph Theory
   - B. Hamilton Paths and Circuits

C. Algorithms (Brute-Force, Nearest-Neighbor, Repetitive Nearest Neighbor)

D. Applications/Puzzles of Graph Theory (Traveling Salesman)

VII. Patterns/Sequences

A. Fibonacci Sequence and the Golden Ratio

B. Golden Ratio Myths and Truths

C. Linear and Geometric Sequences

D. Linear and Geometric Sums

VIII. Symmetry

A. Introduction to Symmetry

B. Rigid Motions

C. Basic Rigid Motions (Reflection, Rotation, Translation, Glide Reflection)

D. Symmetry and Symmetry Types (of finite objects, frieze patterns, and/or wallpaper patterns)

IX. Probability/Combinatorics

A. Sample Spaces and Events

B. Multiplication Rule and Combinations

C. Pascal's Triangle

D. Expected Value