Math 110B : Modern Algebra II

California State University, Sacramento · Department of Mathematics & Statistics

This is the second half of a one year introductory course in Modern Algebra. It is a required course for all students majoring in mathematics. An introduction to groups, rings, fields, and vector spaces will be presented. Some applications may be selected from cryptography, algebraic coding theory, software design, Boolean algebras, electrical circuits, quaternions, finite fields, and constructible numbers, as time permits.

Catalog Description

Continuation of MATH 110A. Note: Topics include: rings and fields. Applications may be selected from lattice, machine, and coding theories. **Graded**: Graded Student. **Units**: 3.0.

Prerequisites

Math 110A

Learning Objectives

The Department of Mathematics & Statistics has a goal in all of its Core Curriculum classes (Math 108, Math 110A/B, and Math 130 A/B) that students be able to effectively communicate matheamtical ideas in written form. This could include clear written explanations of matheamtical ideas as well as constructed matheamtical proofs. The writing allows students to reflect upon their learning and deepen their understanding of the concepts in the courses. It is a useful aspect for understanding the language of mathematics and allows students to express themselves clearly in this language.

Math 110B students will be able to:

- Identify whether a set with multiplicative and additive operations is a ring.
- Identify whether a subset of a ring is an ideal.
- Understand the basic relationship between a commutative ring and the ring of polynomials with coefficients in that ring.
- Prove basic results about homomorphisms of rings, and use the kernel to identify whether a homomorphism is a monomorphism.
- Work fluently with coset arithmetic in factor rings.
- Use the correspondence theorem to identfy ideals of homomorphic images of rings.
- Recognize rings of polynomials as a generalization of the integers, with factorization properties depending on the properties of the ring of coefficients.
- Idenitfy degrees of field extensions.

Course Outline

I. Basic properties of rings (4 weeks)

   A. Defnition, examples, and elementary properties
   B. Subrings
   C. Integral domains, zero divisors, and cancellation
   D. Multiplicative inverses and fields
   E. Characteristic of a ring
   F. Field of quotients of an integral domain

II. Ring of polynomials (2 weeks)

   A. The division algorithm in $F[x]$, with F a field.
   B. Irreducibility in $F[x]$ and the factor theorem.
   C. Results from $\mathbb{Q}[x]$: rational root theorem, factoring in $\mathbb{Q}[x]$ and $\mathbb{Z}[x]$, Eisenstein criterion

III. Ring homomorphisms (3 weeks)

   A. Ideals and factor rings
   B. Fundamental theorem of ring homomorphisms
   C. Maximal and prime ideals
   D. Ideal structure in $F[x]$ and principal ideals.

IV. Vector spaces over a field (2 weeks)

   A. Definitions and examples
   B. Linear dependence and independence
   C. Basis and dimension

V. Extension Fields (3 weeks)

   A. Kronecker's theorem
   B. Algebraic and transcendental elements
   C. Simple extensions
   D. The Fundamental Theorem of Algebra (without proof)