Math 130A : Functions of a Real Variable I

California State University, Sacramento · Department of Mathematics & Statistics

This is a one year upper division course in functions of a real variable that is required of all students majoring in mathematics. The first semester will consist of a rigorous development of the theory of real-valued sequences and continuity and differentiation for functions of one real variable.

Catalog Description

The first half of a one-year upper division course in functions of a real variable. The first semester will consist of a rigorous development of the theory of real-valued sequences and continuity and differentiation for functions of one real variable. **Graded**: Graded Student. **Units**: 3.0.

Prerequisites

Math 32, Math 45, and Math 108

Learning Objectives

The Department of Mathematics & Statistics has a goal in all of its Core Curriculum classes (Math 108, Math 110A/B, and Math 130 A/B) that students be able to effectively communicate matheamtical ideas in written form. This could include clear written explanations of matheamtical ideas as well as constructed matheamtical proofs. The writing allows students to reflect upon their learning and deepen their understanding of the concepts in the courses. It is a useful aspect for understanding the language of mathematics and allows students to express themselves clearly in this language.

Math 130A students will be able to:

- Demonstrate an understanding of the fomral $\epsilon - \delta$ definitions of limits, continuity, differentiability ad integrability and be able to establish basic results using these definitions.
- Prove the fundamental theorems, including the Intermediate Value Theorem, Rolle's Theorem, the Mean Value Theorem, l'Hôspital's Rule, and the Fundamental Theorem of Calculus.
- Apply the results of the major theorems to both standard and nonstandard exercises.
- Demonstrate an understanding of the properties of the real numbers, such as finding an infimum and supremum of specific sets, and using the Archimedian property.
- Distinguish between continuity and uniform continuity and understand the connection between differentiability and continuity.
- Identify whether a function is integrable and show that continuous functions on a closed interval are integrable
- Determine whether a sequence satisfies the Cauchy condition and prove whether a sequence converges or diverges.
- Understand the difference between pointwise convergence and uniform convergence for a sequence of real valued functions.

Course Outline

I. The Real Number System (2 Weeks)

   A. Real numbers as an ordered field
   B. Completeness of the real number system
   C. Archimedean principle
   D. Square roots and absolute value

II. Infinite Sequences (4 Weeks)

   A. $\epsilon - N$ definition of convergence
   B. Bounded sequences
   C. Convergence of $a_n \pm b_n$, $a_n b_n$, and a_n / b_n
   D. Monotone sequences
   E. Subsequences
   F. Cauchy sequences

III. Continuity (4 Weeks)

   A. $\epsilon - \delta$ definition of continuity
   B. Types of discontinuities
   C. Continuity of $f \pm g$, fg, f/g, and $f \circ g$
   D. Maximum-minimum theorem
   E. Intermediate value theorem
   F. Uniform continuity

IV. Differentiation (4 Weeks)

   A. Definition of differentiable
   B. Derivative of x^n, $f \pm g$, fg, and f/g
   C. Chain rule
   D. Derivative and extreme points
   E. Rolle's theorem
   F. Mean value theorem
   G. Statement and applications of L'Hôspital's rule