Math 241A : Foundations of Applied Mathematics

California State University, Sacramento · Department of Mathematics & Statistics

This course covers elementary topics in metric topology and functional analysis designed to give students a good foundation for further study in modern applied mathematics. It presents an introduction to metric spaces, Banach spaces, Hilbert spaces, operators on Hilbert spaces, with applications to differential equations. This course is offered in the Fall.

Catalog Description

Topics from: Hilbert Space Theory, Operators on Hibert Space, Generalized Functions with Applications to Sturm-Liouville Theory and Partial Differential Equations. Note: May be repeated for credit provided topic is not repeated. **Graded**: Graded Student. **Units**: 3.0.

Prerequisites

Math 134 recommended.

Course Outline

I. Review of Linear Algebra and Analysis (1 week)

II. Metric Spaces (3 weeks)

   A. Metric Topology
   B. Continuity
   C. Convergence and Completeness
   D. Compactness
   E. Arzela-Ascoli Theorem
   F. Peano's Existence Theorem for Ordinary Differential Equations

   (Students will learn about convergence and completeness in metric spaces and how to apply these concepts to differential equations.)

III. Elements of Banach Spaces (2 weeks)

   A. Contraction Mapping Theorem
   B. Existence and Uniqueness Theorem for Ordinary Differential Equations

   (Students will learn the elements of Banach spaces and an application of the contraction mapping theorem to proving the existence and uniqueness of solutions of differential equations.)

IV. Elements of Hilbert Spaces (4 weeks)

   A. Orthogonal Expansions
   B. Theorems of Bessel, Parseval, and Riesz-Fischer
   C. Linear Functionals and the Riesz Representation Theorem
   D. Weak Convergence

(Students will learn the elements of Hilbert spaces and its application to the Fourier series expansions of functions.)

V. Operators on a Hilbert Space (4 weeks)

   A. Self-Adjoint Operators

   B. Spectral Theorem

   C. Sturm-Liouville Theory

   D. Partial Differential Equations and Separation of Variables

(Students will learn the elements of operators on a Hilbert space and their applications to solving boundary value problems.)

Reference Book

- A. W. Naylor and G. R. Sell, *Linear Operator Theory in Engineering and Science*, 1982, Springer-Verlag

Additional Reference Books for Math 241A and B

1. R. Abraham, J. E. Marsden, and T. Ratiu, *Manifolds, Tensor Analysis, and Applications, Second Edition*, 1988, Springer-Verlag
2. V. I. Arnold, *Mathematical Methods of Classical Mechanics, Second Edition*, 1989, Springer-Verlag
3. V. I. Arnold, *Ordinary Differential Equations*, 1992, Springer-Verlag
4. S. K. Berberian, *Introduction to Hilbert Space*,1976, Chelsea Publishing Company
5. G. W. Bluman and S. Kumei, *Symmetries and Differential Equations*, 1989, Springer-Verlag
6. J. B. Conway, *A Course in Functional Analysis, Second Edition*, 1997, Springer-Verlag
7. R. Courant and D. Hilbert, *Methods of Mathematical Physics, Vols. I & II*, 1989, John Wiley & Sons
8. P. Dennery and A. Krzywicki, *Mathematics for Physicists*, 1996, Dover Publications
9. J. W. Dettman, *Mathematical Methods in Physics and Engineering*, 1988, Dover Publications
10. G. B. Folland, *Fourier Analysis and Its Applications*, 1992, Brooks/Cole Publishing Company
11. G. B. Folland, *Introduction to Partial Differential Equations, Second Edition*, 1995, Princeton University Press
12. I. M. Gelfand and S. V. Fomin, *Calculus of Variations*, 2000, Dover Publications
13. C. Goffman and G. Pedrick, *First Course in Functional Analysis, Second Edition*, 1983, Chelsea Publishing Company
14. D. H. Griffel, *Applied Functional Analysis*, 2002, Dover Publications
15. R. Grimshaw, *Nonlinear Ordinary Differential Equations*, 1990, Blackwell Scientific Publications
16. J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, 1983, Springer-Verlag
17. R. B. Guenther and J. W. Lee, *Partial Differential Equations of Mathematical Physics and Integral Equations*, 1996, Dover Publications
18. K. E. Gustafson, *Introduction to Partial Differential Equations and Hilbert Space Methods, Third Edition*, 1999, Dover Publications

19. J. K. Hale, *Ordinary Differential Equations*, 1969, John Wiley & Sons

20. J. Hale and H. Kocak, *Dynamics and Bifurcations*, 1991, Springer-Verlag

21. M. W. Hirsch and S. Smale, *Differential Equations, Dynamical Equations, and Linear Algebra*, 1974, Academic Press

22. S. S. Holland, Jr., *Applied Analysis by the Hilbert Space Method*, 1990, Marcel Dekker

23. W. Hurewicz, *Lectures on Ordinary Differential Equations*, 1990, Dover Publications

24. O. D. Kellogg, *Foundations of Potential Theory*, 1953, Dover Publications

25. A. N. Kolmogorov and S. V. Fomin, *Elements of the Theory of Functions and Functional Analysis*, 1999, Dover Publications

26. A. N. Kolmogorov and S. V. Fomin, *Introductory Real Analysis*, 1975, Dover Publications

27. E. Kreyszig, *Introductory Functional Analysis with Applications*, 1989, John Wiley & Sons

28. E. R. Lorch, *Spectral Theory*, 1962, Oxford University Press

29. D. G. Luenberger, *Optimization by Vector Space Methods*, 1969, John Wiley & Sons

30. J. Macki and A. Strauss, *Introduction to Optimal Control Theory*, 1982, Springer-Verlag

31. G. L. Naber, *Topology, Geometry, and Gauge Fields: Foundations*, 1997, Springer-Verlag

32. A. W. Naylor and G. R. Sell, *Linear Operator Theory in Engineering and Science*, 1982, Springer-Verlag

33. A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, 1983, Springer-Verlag

34. F. Riesz and B. Sz.-Nagy, *Functional Analysis*, 1990, Dover Publications

35. H. L. Royden, *Real Analysis, Second Edition*, 1968, The Macmillan Company

36. W. Rudin, *Functional Analysis, Second Edition*, 1991, McGraw-Hill

37. W. Rudin, *Principles of Mathematical Analysis, Third Edition*, 1976, McGraw-Hill

38. W. Rudin, *Real and Complex Analysis, Third Edition*, 1987, McGraw-Hill

39. H. Sagan, *Boundary and Eigenvalue Problems in Mathematical Physics*, 1989, Dover Publications

40. H. Sagan, *Introduction to the Calculus of Variations*, 1992, Dover Publications

41. D. A. Sanchez, *Ordinary Differential Equations and Stability Theory: An Introduction*, 1968, W. H. Freeman and Company

42. L. Schwartz, *Mathematics for the Physical Sciences*, 1966, Addison-Wesley Publishing Company

43. G. F. Simmons, *Differential Equations with Applications and Historical Notes*, 1972, McGraw-Hill

44. G. F. Simmons, *Introduction to Topology and Modern Analysis*, 1963, McGraw-Hill

45. I. Stakgold, *Green's Functions and Boundary Value Problems*, 1979, John Wiley & Sons

46. G. P. Tolstov, *Fourier Series*, 1976, Dover Publications

47. F. Verhulst, *Nonlinear Differential Equations and Dynamical Systems, Second Edition*, 1996, Springer-Verlag

48. H. F. Weinberger, *A First Course in Partial Differential Equations with Complex Variables and Transform Methods*, 1995, Dover Publications

49. R. Weinstock, *Calculus of Variations With Applications to Physics and Engineering*, 1974, Dover Publications

50. E. Zauderer, *Partial Differential Equations of Applied Mathematics, Second Edition*, 1989, John Wiley & Sons