# Math 241B : Topics in Applied Mathematics

California State University, Sacramento · Department of Mathematics & Statistics

With the permission of the department, this course (offered with different topics) may be taken for credit more than once. This course is offered in the Spring.

This is a topics in modern applied mathematics course designed to present a variety of methods of modern analysis and differential equations useful in solving problems originating in science and engineering. The topics, chosen from modern fields of applied mathematics, may vary depending upon the interests of the instructor. Topics may be selected from the following areas: **Generalized Functions** (Fourier Transforms, Green's Functions), **Calculus of Variations** (Euler-Lagrange Equations, Isoperimetric Inequality, Eigenvalue Problems), **Topics in Functional Analysis** (Hahn-Banach Theorem, Bounded and Continuous Operators, Weak* Convergence, Sobolev Spaces), **Dynamical Systems** (Autonomous and Nonautonomous Systems, Stability, Liapunov Functions, Poincare-Bendixson Theorem), **Integral Equations** (Hilbert-Schmidt Theory, Fredholm Theory), **Potential Theory** (Harmonic Functions, Green's Functions), and **Optimal Control Theory** (Controllability, Bang-Bang Principle, Pontryagin Maximum Principle).

## Catalog Description

Continuation of Math 241A with topics: Calculus of Variations, Functional Analysis, Dynamical Systems, Integral Equations, Sobolev Spaces, Fourier Analysis, Potential Theory, and Optimal Control Theory. Note: May be taken twice with approval of the graduate coordinator. **Graded**: Graded Student. **Units**: 3.0.

## Prerequisites

Math 241A.

## Course Outline

   I. Review of Advanced Calculus and Analysis (1 week)

  II. Calculus of Variations: The Elements (2 weeks)

   A. Function Spaces and Functionals

   B. The Gateaux Differential

   C. Euler-Lagrange Equations

   (Students will learn the theoretical underpinnings of variational methods.)

 III. Calculus of Variations: Further Developments and Applications (3 weeks)

   A. Second Variation

   B. Hamilton's Principle

   C. Brachistochrone Problem, Isoperimetric Inequality, and Geodesics on Surfaces

   D. Variational methods applied to eigenvalue problems

   (Students will apply variational methods to solve some classic optimization problems in physics and applied mathematics.)

  IV. Dynamical Systems: Linear Theory (2 1/2 weeks)

A. Fundamental Solutions

B. Autonomous Linear Systems and Phase Portraits

C. Critical Points and Stability

(Students will learn to solve linear systems of differential equations, to locate the critical points of the system, and analyze the stability of the system near a critical point.)

V. Dynamical Systems: Nonlinear Theory (3 1/2 weeks)

A. Autonomous Nonlinear System

B. Linearizing a Nonlinear System at a Critical Point

C. Stability

D. Liapunov Functions

E. Poincare-Bendixson Theorem

(Students will learn to analyze the stability of a nonlinear dynamical system using both linearization and Liapunov functions, and how to apply the Poincare-Bendixson theorem to determine the existence of a closed orbit.)

VI. Dynamical Systems: Applications (2 weeks)

A. Conservative Force Fields and Elliptical Planetary Orbits

B. Hamiltonian Mechanics

C. Volterra-Lotka Predator-Prey Equations

(Students will apply techniques of dynamical systems to analyze solutions of problems from applied mathematics and related fields.)

## Reference Books

- H. Sagan, *Introduction to the Calculus of Variations*, 1992, Dover Publications

- M. W. Hirsch and S. Smale, *Differential Equations, Dynamical Equations, and Linear Algebra*, 1974, Academic Press

## Additional Reference Books for Math 241A and B

1. R. Abraham, J. E. Marsden, and T. Ratiu, *Manifolds, Tensor Analysis, and Applications, Second Edition*, 1988, Springer-Verlag

2. V. I. Arnold, *Mathematical Methods of Classical Mechanics, Second Edition*, 1989, Springer-Verlag

3. V. I. Arnold, *Ordinary Differential Equations*, 1992, Springer-Verlag

4. S. K. Berberian, *Introduction to Hilbert Space*, 1976, Chelsea Publishing Company

5. G. W. Bluman and S. Kumei, *Symmetries and Differential Equations*, 1989, Springer-Verlag

6. J. B. Conway, *A Course in Functional Analysis, Second Edition*, 1997, Springer-Verlag

7. R. Courant and D. Hilbert, *Methods of Mathematical Physics, Vols. I & II*, 1989, John Wiley & Sons

8. P. Dennery and A. Krzywicki, *Mathematics for Physicists*, 1996, Dover Publications

9. J. W. Dettman, *Mathematical Methods in Physics and Engineering*, 1988, Dover Publications

10. G. B. Folland, *Fourier Analysis and Its Applications*, 1992, Brooks/Cole Publishing Company

11. G. B. Folland, *Introduction to Partial Differential Equations, Second Edition*, 1995, Princeton University Press

12. I. M. Gelfand and S. V. Fomin, *Calculus of Variations*, 2000, Dover Publications

13. C. Goffman and G. Pedrick, *First Course in Functional Analysis, Second Edition*, 1983, Chelsea Publishing Company

14. D. H. Griffel, *Applied Functional Analysis*, 2002, Dover Publications

15. R. Grimshaw, *Nonlinear Ordinary Differential Equations*, 1990, Blackwell Scientific Publications

16. J. Guckenheimer and P. Holmes, *Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields*, 1983, Springer-Verlag

17. R. B. Guenther and J. W. Lee, *Partial Differential Equations of Mathematical Physics and Integral Equations*, 1996, Dover Publications

18. K. E. Gustafson, *Introduction to Partial Differential Equations and Hilbert Space Methods, Third Edition*, 1999, Dover Publications

19. J. K. Hale, *Ordinary Differential Equations*, 1969, John Wiley & Sons

20. J. Hale and H. Kocak, *Dynamics and Bifurcations*, 1991, Springer-Verlag

21. M. W. Hirsch and S. Smale, *Differential Equations, Dynamical Equations, and Linear Algebra*, 1974, Academic Press

22. S. S. Holland, Jr., *Applied Analysis by the Hilbert Space Method*, 1990, Marcel Dekker

23. W. Hurewicz, *Lectures on Ordinary Differential Equations*, 1990, Dover Publications

24. O. D. Kellogg, *Foundations of Potential Theory*, 1953, Dover Publications

25. A. N. Kolmogorov and S. V. Fomin, *Elements of the Theory of Functions and Functional Analysis*, 1999, Dover Publications

26. A. N. Kolmogorov and S. V. Fomin, *Introductory Real Analysis*, 1975, Dover Publications

27. E. Kreyszig, *Introductory Functional Analysis with Applications*, 1989, John Wiley & Sons

28. E. R. Lorch, *Spectral Theory*, 1962, Oxford University Press

29. D. G. Luenberger, *Optimization by Vector Space Methods*, 1969, John Wiley & Sons

30. J. Macki and A. Strauss, *Introduction to Optimal Control Theory*, 1982, Springer-Verlag

31. G. L. Naber, *Topology, Geometry, and Gauge Fields: Foundations*, 1997, Springer-Verlag

32. A. W. Naylor and G. R. Sell, *Linear Operator Theory in Engineering and Science*, 1982, Springer-Verlag

33. A. Pazy, *Semigroups of Linear Operators and Applications to Partial Differential Equations*, 1983, Springer-Verlag

34. F. Riesz and B. Sz.-Nagy, *Functional Analysis*, 1990, Dover Publications

35. H. L. Royden, *Real Analysis, Second Edition*, 1968, The Macmillan Company

36. W. Rudin, *Functional Analysis, Second Edition*, 1991, McGraw-Hill

37. W. Rudin, *Principles of Mathematical Analysis, Third Edition*, 1976, McGraw-Hill

38. W. Rudin, *Real and Complex Analysis, Third Edition*, 1987, McGraw-Hill

39. H. Sagan, *Boundary and Eigenvalue Problems in Mathematical Physics*, 1989, Dover Publications

40. H. Sagan, *Introduction to the Calculus of Variations*, 1992, Dover Publications

41. D. A. Sanchez, *Ordinary Differential Equations and Stability Theory: An Introduction*, 1968, W. H. Freeman and Company

42. L. Schwartz, *Mathematics for the Physical Sciences*, 1966, Addison-Wesley Publishing Company

43. G. F. Simmons, *Differential Equations with Applications and Historical Notes*, 1972, McGraw-Hill

44. G. F. Simmons, *Introduction to Topology and Modern Analysis*, 1963, McGraw-Hill

45. I. Stakgold, *Green's Functions and Boundary Value Problems*, 1979, John Wiley & Sons

46. G. P. Tolstov, *Fourier Series*, 1976, Dover Publications

47. F. Verhulst, *Nonlinear Differential Equations and Dynamical Systems, Second Edition*, 1996, Springer-Verlag

48. H. F. Weinberger, *A First Course in Partial Differential Equations with Complex Variables and Transform Methods*, 1995, Dover Publications

49. R. Weinstock, *Calculus of Variations With Applications to Physics and Engineering*, 1974, Dover Publications

50. E. Zauderer, *Partial Differential Equations of Applied Mathematics, Second Edition*, 1989, John Wiley & Sons