Foundations of Relational Realism

Relational Realism and Quantum Geometric Phases

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Participants:

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Meeting summary:

This is the second meeting of our 2013 research program: Experimental Application of the Relational Realist Formalism: A Topological, Sheaf-Theoretic Explication of Quantum Geometric Phases By Analysis of Experimental Data on the Aharonov-Bohm Effect, the Pancharatnam Phase, and the Quantum Hall Effect, Toward a Unified Interpretation.

After nearly a century of development, the central conceptual and interpretive problems in quantum mechanics still remain unsettled, even in the wake of marked improvements in technology and experimental methodology. Among these now infamous and interrelated problems are: [1] the problem of measurement; [2] quantum nonlocality; [3] the coherent integration of quantum and classical physical theories.

In our current project, we demonstrated how all three of these difficulties can be properly understood as aspects of a single problem: the absence in quantum mechanics of a formal means of relating local to global in an extensive continuum. While this problem is most popularly exemplified in the incompatibility of quantum mechanics and the general theory of relativity, we demonstrated that its proper solution lies first in recognizing the centrality of local-global relations in all three of the aforementioned problems; and second, recognizing that the overall genesis of difficulty is the presumption of a fundamentally metrical theory of extension grounded in a set-theoretic structure. While this convention has clearly proven fruitful as a conceptual framework for formal physics, the latter's evolutionary leap in the early 20th century with the advent of quantum theory and general relativity has rendered explicit its limitations—viz. its vulnerabilities to paradoxes, singularities, and infinites. Since modern theoretical physics is essentially mathematical physics, we have today two possible pathways forward: [1] to incorporate these vulnerabilities into our physical theories by proposing their *physical* instantiation (viz. 'physical' singularities as black hole cores, or physical paradoxes of counterfactual states ('real' Schrödinger cats); or [2] to clearly identify and overcome the limitations of the presupposed metrical, set-theoretic framework by which extension is fundamentally understood, so that such physical instantiations of mathematical singularities and paradoxes can be avoided.

In our forthcoming volume, *Foundations of Relational Realism: A Topological Approach to Quantum Mechanics*, Lexington Books (2013), we charted out this second pathway by first identifying the central conceptual deficiency of the conventional metrical, set-theoretic understanding of extension: *That it characterizes objects as fundamental to relations.* Prior to quantum mechanics, this deficiency went unnoticed; but since it is a signature feature of quantum mechanics that it definitively proscribes specifying the existence of objects *in abstraction* from their relations, the attempt to depict quantum mechanical extensiveness as fundamentally metrical—again, such that objects are fundamental to their relations—is doomed from the beginning.

The solution we proposed is to delve beneath this conventional set theoretic framework and uncover a more substrative category theoretic framework where extension is fundamentally topological rather than metrical. In this way, fundamental quanta are properly defined as 'units of logico-physical relation' rather than 'units of physical relata.' By this framework, objects are always understood *as* relata, such that objects and relations are coherently defined as mutually implicative.

Our *Foundations of Relational Realism*, along with a second volume, *Physics and Speculative Philosophy: Potentiality, Actuality, and Process* forthcoming from Ontos-Verlag, represent the capstones of our current Fetzer funded project, along with a number of published papers and conference presentations. With this foundational work in place—a rigorous philosophical and mathematical formalism applicable to quantum mechanics specifically, and to natural philosophy more generally—our next step is to demonstrate the *experimental applicability* of this formalism—viz. its advantages both in terms of prediction and interpretation of data. To that end, for 2013, we propose to apply our relational realist formalism to the task of explicating the well-known but poorly understood problem of quantum geometric phases.

We will do so by analysis of experimental data on the Aharonov-Bohm Effect, the Pancharatnam Phase, and the Quantum Hall Effect, toward a unified interpretation of all three.

> "Knowledge of the classical electromagnetic field acting locally on a particle is not sufficient to predict its quantum-mechanical behavior... For a long time it was believed that ***A*** [the vector potential] was not a 'real' field... There are phenomena involving quantum mechanics which show that in fact ***A*** is a 'real' field in the sense that we have defined it... ***E*** and ***B*** [electromagnetic fields] are slowly disappearing from the modern expression of physical laws; they are being replaced by ***A*** [the vector potential] and φ[the scalar potential]."
>
> Richard Feynman
> *The Feynman Lectures on Physics*

Geometric Phases in Quantum Mechanics: A Topological, Sheaf Theoretic Approach

> *We believe that the concept of a geometric phase, repeating the history of the group concept, will eventually find so may realizations and applications in physics that it will repay study for its own sake, and become part of the lingua franca.*
>
> *A. Shapere and F. Wilczek*

In a groundbreaking paper published in the mid-1980's, Sir M. Berry discovered that a quantum system undergoing a slowly evolving (adiabatic) cyclic evolution retains a "memory" of its motion after coming back to its original physical state. This "memory" is expressed by means of a complex phase factor in the wave-function of the system, called Berry's phase or the geometric phase. The main idea is that a quantum system in a slowly changing environment displays a *history dependent geometric effect*: When the environment returns to its original state, the system also does, but for an additional phase.

The Berry phase is a complex number of modulus one and is experimentally observable. The two most important properties of the geometric phase are [1] that it is a statistical object, and [2] it can be measured only relatively. Thus it becomes observable by comparing the evolution of two distinct statistical ensembles of systems through their interference pattern. The Berry phase is called 'geometric' because it depends *solely on the topology of the pathway along which the system evolves and neither on the temporal metric duration of the evolution nor on the dynamics that is applied to the system*.

After Berry's initial experimental discovery of the geometric phase, it has been demonstrated that it also exists for non-adiabatic evolutions and even for *sequences* of measurements. All these experimental observations of the geometric phase phenomenon, including the Aharonov-Bohm, Pancharatnam, and Quantum Hall effects—as well as features of molecular and nuclear spectra, vortices occurring in superfluid helium, and even chemical reactions—help to emphasize its purely topological nature. In this way, the geometric phase observable initiated a tremendous surge of experimental research across a diverse range of physics disciplines establishing surprising connections among a number of apparently disparate phenomena—again, viz. the Aharonov-Bohm effect, its molecular physics analogue by Mead and Truhlar, polarization optics effects, and the Quantum Hall effect. Thus, we expect that a unified topological approach to the study, description and interpretation of all these experimentally well-established geometric phase phenomena will shed light on their nature, lead to predictions of new effects of a similar type, and even provide a paradigm for applications in areas beyond the strict domain of natural sciences.

The novel topological approach to quantum mechanics we present in *Foundations of Relational Realism*, in the context of our overall Fetzer research project and its many publications, provides a perfect candidate for evaluating consistently the experimental data of geometric phases observed in all the above experiments. Such an evaluation would demonstrate our work's applicability to diverse quantum phenomena, explicated via the lens of a unifying topological theoretical framework, which is based on the powerful modern mathematical methods of category theory and sheaf theory.

The application of our topological formalism to the analysis of experimental data, carrying with it an associated unifying realistic interpretation of geometric phase phenomena, thus constitutes a natural continuation and experimental substantiation of our previous research program. The basic principle underlying the crucial explanatory role of this approach in relation to geometric phases is the following: *Whenever a global physical system is partitioned into local parts and one attempts to describe a subsystem in isolation, the influence and connectivity of the others is manifested through geometric phase effects of a pure topological origin.*

The pervasiveness throughout the natural sciences of this part-whole methodology accounts for the ubiquity of the geometric phase effect, but not its explanation. We propose to solve this problem via our realistic mereotopological formalism and conceptual framework. To that end, the precise mathematical notion which captures the topological origin of geometric phases is the *anholonomy* of a connection on a sheaf modeling the structure of events. The notion of a sheaf connection provides the technical means for describing the process of parallel transport of the state vector of a system along a curve on the space of the control variables. The particular transformation undergone by the state vector when it is parallel-transported along a closed curve is called the *anholonomy of the sheaf connection*. Thus, the anholonomy describes the state vector transformation induced by cyclic changes (loops) in the controlling variables. A simple intuitive example from macroscopic geometry is provided by the rotation of a vector when parallel-transported along a closed path on a curved surface. The topological nature of the effect is realized by the fact that if the parallel transport of a vector takes place along a closed path on a plane, the vector comes back unchanged; whereas if it takes place along a closed path on a sphere it gets rotated by an angle, which is measured precisely by the anholonomy of the connection.

Sample of Experiments Analyzed

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