Getting Something Out of Nothing: Implications for a Future Information Theory Based on Vacuum Microtopology[1]

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Abstract

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Contemporary theoretical physicists H. S. Green and David R. Finkelstein have recently advanced theories depicting space-time as a singular limit, or condensate, formed from fundamentally quantum micro topological units of information, or process (denoted respectively by 'qubits,' or 'chronons.') H. S. Green (2000) characterizes the manifold of space-time as a parafermionic statistical algebra generated fundamentally by qubits. David Finkelstein (2004a-c) models the space-time manifold as singular limit of a regular structure represented by a Clifford algebra, whose generators γ^{α} represent 'chronons,' i.e., elementary quantum processes. Both of these theories are in principle experimentally testable. Green writes that his parafermionic embeddings "hav[e] an important advantage over their classical counterparts [in] that they have a direct physical interpretation and their parameters are in principle observable." (166) David Finkelstein discusses in detail unique empirical ramifications of his theory in (2004b,c) which most notably include the removal of quantum field-theoretic divergences. Since the work of Shannon and Hawking, associations with entropy, information, and gravity emerged in the study of Hawking radiation. Nowadays, the theories of Green and Finkelstein suggest that the study of space-time lies in the development of technologies better able to probe its microtopology in controlled laboratory conditions.

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I. Introduction

According to Herbert Sydney Green (2000), the role played by quantum theory with respect to information theory is anticipated in the following passage:

> In the early years of the development of the quantized theories, the principal innovation was that the field variables were treated as matrices, but it also became important to derive the differential equations from the Principle of Least Action in order to obtain an unambiguous formulation of the commutation relations satisfied by the matrices. The present approach to quantized field theory…is more strongly influenced by information theory. *As a consequence of quantization, a field is ultimately interpreted as providing a representation of the transmission of information through particles of the same type but possibly different momenta.* (italics added, 116)

Green presents a unified theory, comprising all areas of field theory and gravitation, on the one hand, to the biophysics of neurophysiology and brain research, on the other, based fundamentally on an extended notion of the qubit. Of primary interest here is to investigate how the space-time manifold is constituted fundamentally by such units of information, described in his theory in terms of embeddings in a parafermionic algebra. These parafermionic embeddings "have a direct physical interpretation and their parameters are in principle observable." (166)

In a closely parallel fashion, David Finkelstein (2001, 2004a-c) models the space-time manifold as a singular limit of a regular structure represented by a Clifford algebra, whose generators γ^{α} represent 'chronons,' or elementary quantum processes. His latest research is part of an ongoing development originating in the *Space-Time Code* papers (1969), which, like in the case of Green, formed the basis of an "attempt to express field theory in terms of q bits or chronons." (2001, 9). Finkelstein (2004b,c) presents some potentially observable consequences of his theory.

I will discuss the ramifications of the above theories, which share the ontological intuition of conceiving space-time itself as fundamentally generated or derived from an underlying microtopology of fundamental quantum processes of information. The empirical tests discussed by Green and Finkelstein raise compelling questions for future information-based technologies. Prior to doing so, however, key aspects of Green and Finkelstein's theories must be brought to light.

II. H.S. Green's Extended Qubits and Parafermionic Embeddings

An *extended qubit* is represented by a class of matrices $\mathbf{M} \subseteq \mathbf{C}^2$ [(4)] obeying unipotency and unit trace:

Defn. (Extended Qubit) For any $M \in \mathbf{M}$: (a) $M^2 = M$ and (b) tr[M] = 1. Expressed component-wise:

(a) $\sum_{k=1}^{N} M_{ik} M_{kj} = M_{ij}$ (b) $\sum_{k=1}^{N} M_{kk} = 1$, where $N = \dim(M) \geq 2$.[5]

The solutions to constraints (a) & (b) partition **M** into the equivalence classes of Hermitian[6], pseudo-Hermitian[7], and real-valued matrices[8]. These three classes represent three different *kinds* of qubits. These kinds are characterized physically by the representation of information in ordinary QM (in the inertial frame of the observer),

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[(4)] $\mathbf{C}^2$ of course is the Hilbert Space: $\mathbf{C} \times \mathbf{C}$, where: $\mathbf{C}$ is the complex numbers.

[5] There are cases of course when the dimension exceeds 2 (for example, in the case of Dirac gamma matrices of dimension 4.) Though the number of independent parameters characterizing the qubit remains invariant and independent of its particular representation in any given full matrix algebra..

[6] I.e., any matrix A where: $A_{jk}^* = A_{kj}$ (the complex conjugate of A = the transpose of A.)

[7] I.e., any matrix A such that, corresponding to A is another (Hermitian) matrix C which is idempotent (C^2 = Id) and CA is Hermitian.

[8] I.e., $A \in \mathbf{R}^2$.

information adopted in other inertial frames, and information derived from distant sources. (34)

The first (Hermitian) class is what one normally thinks of as a qubit in standard QM since it has the typical Pauli spinor representation: $Q(\xi) = {}^1/_2\{\mathrm{Id} + \boldsymbol{\xi}\bullet\boldsymbol{\sigma}\}$ (where: Id is the 2×2 identity matrix, ξ is a 3D spatial vector of unit norm, and $\boldsymbol{\xi}\bullet\boldsymbol{\sigma}$ is its expansion in the Pauli matrix basis[9].) Since the relativity group[10] of standard QM is Galilean, it makes sense to think of $Q(\xi)$ as representing information in the observer's 'proper' inertial frame of reference (IFR).

The second anti-Hermitian class, on the other hand, extends to applications in special relativity, ascribed by Green as the interpretation of a qubit in quantum field theory (i.e., Lorenz-invariant QM.) Such a qubit has representation:

$Q(\omega) = {}^1/_2\{\mathrm{Id} + \boldsymbol{\omega}\bullet\boldsymbol{\rho}\}$ (where: $\boldsymbol{\omega}\bullet\boldsymbol{\rho} = \omega_0\rho_0 - \omega_1\rho_1 - \omega_2\rho_2$ and $\omega_0^2 - \omega_1^2 - \omega_2^2 = 1$ and the matrices ρ_1, ρ_2 are anti-Hermitian.) In locally flat space-time, the Lorentz Group describes how two or more IFRs transform, hence $Q(\omega)$ represents information adopted on different IFRs from that of the observer.

The third (real) class is found in applications in curved space-time (in particular, deSitter Space) denoted as a *space-like* qubit.[11] Though deSitter space (projectively) contains all significant cases of flat space-times, the instances involving use of

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[9] $\boldsymbol{\xi}\bullet\boldsymbol{\sigma} = \xi_1\sigma_1 + \xi_2\sigma_2 + \xi_3\sigma_3$ where the subscripts 1,2,3 refer, of course, to the *x*, *y*, *z* components of $\boldsymbol{\xi}$ and Pauli matrix σ, respectively.

[10] The group of all dynamical symmetries invariant under Galilean transformations.

[11] DeSitter space is a topologically closed and spherically symmetric manifold which can be conveniently thought of in terms of a four-dimensional projective geometry. Its one-dimensional closed subspaces of rays of infinite extent describe timelike trajectories in the space, while its curved 3D subspaces are of finite radius R and describe the closure of set of all spacelike separated points. $Q(\eta)$ co-vary with respect to transports of points in such space-like 3D 'great circles', hence their space-like association. In the $R \rightarrow \infty$ limit deSitter space becomes the Minkowski space-time of special relativity.

Riemannian geometry in general relativity don't apply.[12] Such a qubit has representation: $Q(\eta) = {}^1/_2\{\text{Id} + \boldsymbol{\eta}\bullet\boldsymbol{\tau}\}$ (where: $\boldsymbol{\eta}\bullet\boldsymbol{\tau} = -\eta_0\tau_0 + \eta_1\tau_1 + \eta_2\tau_2$ and $-\eta_0^2 + \eta_1^2 + \eta_2^2 = 1$ and the matrices τ_1, τ_2 are anti-Hermitian.) DeSitter space, for extremely long ranges, (i.e. for deSitter radius $R \approx$ intergalactic distances) provides the simplest approximation to the global properties of a dynamically varying metric $g_{\mu\nu}(x)$ described in general relativity. Hence $Q(\eta)$ represents information derived from distant sources.

Introducing his parafermion representation of Lie algebras, the statistical machinery governing how qubits combine, the impetus of constructing such a statistics is guided the intuition that a space-time point (x^μ) should be interpreted as an *event* wherein a neutral particle is emitted or absorbed. Emission and absorption, represented by respective parafermion elements ς_0, ς, while the particle's geodesic path can be represented projectively by the join: $x^\mu(\varsigma_0)\vee x'^\mu(\varsigma)$, where x, x' are the space-time points corresponding to detection/absorption events.(147) Then, the manifold of space-time itself may be characterized in terms of a parafermionic statistical algebra Σ fundamentally generated by qubits.[13] Green describes this procedure as 'quantal embedding.' In one case he succeeds in embedding Riemannian geometry[14] into Σ wherein the metric, for example, takes on the form:

$$g_{\mu\nu} \equiv \sum_{r=1}^{2s} \bar{\varsigma}_\mu^{(r)} \otimes \varsigma_\nu^{(r)} \qquad \text{(II.1)}$$

[12] Green takes up the general relativistic case in chapter 7.

[13] "The quantum mechanics of systems with large numbers of interacting particles…can be given a formulation in which the elements…are represented by fermions or parafermions, *and thus in terms of qubits*." (108, italics added)

[14] "The *quantal* embedding of the Riemannian space [has] 'coordinates' of the embedding space [which] are…parameters of the group of transformations connecting different ς_0, ς." (163)

where: $\bar{\varsigma}_{\mu}, \varsigma_{\nu} \in \Sigma$, $\dim\Sigma = 2s$, and the bar superscript denotes the Majorana adjoint.

Green describes his quantal embeddings as having a direct physical interpretation which make their parameters observable, in principle. (166) For instance, the geodesics $x^{\mu}(\varsigma_0) \vee x'^{\mu}(\varsigma)$ apply to the trajectories of *neutral* particle propagation, whether photons or neutrinos. "[I]t is not clear that a physical geometry constructed from the observation of neutrinos would be the same as that derived from the observation of light, but an informationally based theory could well provide…indication of differences which in the future could be detected experimentally." (147)

III. Finkelstein's Chronons and their Clifford Algebraic Characterizations

Finkelstein (1996, 2001, 2004a-c) presents a unification of field theories (quantum and classical) and space-time theory based fundamentally on *finite* dimensional algebraic structures, and on a regularization procedure fundamentally involving group-theoretic simplification.[15] The choice of the Clifford Algebra[16] is motivated by two fundamental reasons:

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[15] I.e., expanding into a group with no invariant subgroups, which among other things stabilizes its Lie algebras. This technique of simplification and regularization has its origins in the work of Inonou & Wigner (1952) and I. E. Segal (1951). For instance, Inonu and Wigner show that in the expansion from the (non-simple) Galilean group to the (simple) Lorenz group, the latter's Lie algebra is stable, while the former is not. These powerful techniques of simplification and regularization can be viewed as necessary criteria for fundamental physical theories, present and future. Regularization cures theories of pathological singularities. "Segal…stimulate[s] the present work…which seem to lead… to a finite quantum theory and a quantum space-time, goals of some physicists since the formation of quantum theory. They produce a theory with a simple group, having the prior theory as a limiting case and having nearly the same continuous symmetries." (2001, 2)

[16] The topic of Clifford algebras and their applications has enjoyed widespread attention in mathematical physics, (Bolinder (1987)). Aside from reasons 1.) and 2.) mentioned above, which are motivated by specifically *physical* notions, there are the metatheoretic features of regularization, group simplicity, and Lie algebraic stability that motivates Finkelstein's research program in its selection of Clifford algebras.

1. The typically abstract (adjoint-based) algebraic characterizations of quantum dynamics (whether C^*, Heisenberg, etc.) just represent how actions can be combined (in series, parallel, or reversed) but omit space-time fine structure.[17] On the other hand, a Clifford algebra can express a quantum space-time. (2001, 5)

2. Clifford statistics[18] for chronons adequately expresses the distinguishability of events as well as the existence of half-integer spin. (2001, 7)

The first reason entails that the prime variable is not the space-time field, as Einstein stipulated, but rather the dynamical law. That is to say, "the dynamical law [is] the only dependent variable, on which all others depend.[19]" (2001, 6) The "atomic" quantum dynamical unit (represented by a generator γ^{α} of a Clifford algebra) is the *chronon* χ, with a closest classical analogue being the tangent or cotangent vector, (forming an 8-dimensional manifold) and *not* the space-time point (forming a 4-dimensional manifold).

Applying Clifford statistics to dynamics is achieved via the (category) functors ENDO, SQ which map the mode space[20] X of the chronon χ, to its operator algebra (the algebra of endomorphisms A on X) and to its spinor space S (the statistical composite of all chronons transpiring in some experimental region.) (2001, 10). The action of ENDO, SQ producing the Clifford algebra *CLIFF*, representing the global dynamics of the chronon ensemble is depicted in the following commutative diagram:

There exist a variety of different axiomatic characterizations of Clifford algebra, Finkelstein's is most heavily reliant on Hestenes and Sobczyk's (1984) treatment.

[17] The space-time structure must are supplied by classical structures, prior to the definition of the dynamical algebra. (2001, 5)

[18] I.e., the simplest statistics supporting a 2-valued representation of S_N, the symmetry group on N objects.

[19] This comprises a general and central notion in Finkelstein's research, what he terms as "praxism," in contrast with "ontism." A praxic characterization begins with a notion of elementary actions and dynamical law, showing that a notion of "state" is derivative. Ontism works in the opposite direction, taking state as the primitive and deriving elementary dynamics in terms of mappings between states. For a detailed discussion, see chapters 1 – 4 in Finkelstein (1996.)

[20] The mode space is a kinematic notion, describing the set of all possible modes for a chronon χ, the way a state space describe the set of all possible states for a state φ in ordinary quantum mechanics.

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\begin{array}{ccc}
X & \xrightarrow{\quad ENDO \quad} & A = \mathrm{ENDO}(X) \\
\downarrow{\scriptstyle SQ} & & \downarrow{\scriptstyle SQ} \\
S & \xrightarrow{\quad ENDO \quad} & C\mathit{LIFF}
\end{array}
$$

Analogous to Green's embedding of the space-time geometry into a paraferminionic algebra of qubits, Finkelstein shows that a Clifford statistical ensemble of chronons can factor as a Maxwell-Boltzmann ensemble of Clifford subalgebras. This in turn becomes a Bose-Einstein aggregate in the $N \to \infty$ limit (where N is the number of factors.) This Bose-Einstein aggregate condenses into an 8-dimensional symplectic manifold M which is isomorphic to the tangent bundle of space-time. Moreover, M is a *Clifford manifold*, i.e. a manifold provided with a Clifford ring: $C(M) = C_0(M) \oplus C_1(M) \oplus \ldots \oplus C_N(M)$ (where: $C_0(M)$, $C_1(M)$,…,$C_N(M)$ represent the scalars, vectors,…, N-vectors on the manifold.) For any tangent vectors $\gamma^{\mu}(x)$, $\gamma^{\nu}(x)$ on (Lie algebra dM) then:

$$\gamma^{\mu}(x) \circ \gamma^{\nu}(x) = g^{\mu\nu}(x) \qquad \text{(III.1)}$$

where: $\circ$ is the scalar product. (2004, 43) Hence the space-time manifold is a singular limit of the Clifford algebra representing the global dynamics of the chronons in an experimental region.

Observable consequences of the theory are discussed in the model of the oscillator (2004c). Since the dynamical oscillator undergirds much of the framework of contemporary quantum theory, especially quantum field theory, the (generalized) model oscillator constructed via group simplification and regularization is isomorphic to a dipole rotator in the orthogonal group O(6N) (where: $N = l(l + 1) >> 1$). In other words, a *finite* quantum mechanical oscillator results, bypassing the ultraviolet and infrared divergences that occur in the case of the standard (infinite dimensional) oscillator applied to quantum field theory. In place of these divergences, are "soft" and "hard" cases, respectively representing maximum potential energy unable to excite one quantum of momentum, and maximum kinetic energy being unable to excite one quantum of position. "These [cases]…resemble [and] extend the original ones by which Planck obtained a finite thermal distribution of cavity radiation. Even the 0-point energy of a similarly regularized field theory will be finite, and can therefore be physical." (2004c, 12)

In addition, such potentially observable extreme cases modifies high and low energy physics, as "the simplest regularization leads to interactions between the previously uncoupled excitation quanta of the oscillator…strongly attractive for soft or hard quanta." (2004c, 19) Since the oscillator model quantizes and unifies time, energy, space, and momentum, on the scale of the Planck power (10^{51} W) time and energy can be interconverted.

Moreover, in such extreme cases, equipartition and Heisenberg Uncertainty is violated. The uncertainty relation for the soft and hard oscillators read, respectively:

$$(\Delta L_1)^2(\Delta L_2)^2 \geq \frac{\hbar^2}{4}\langle L_3\rangle^2{}_{|L_1\approx 0\rangle} \approx 0 \Rightarrow \Delta p\Delta q << \frac{\hbar}{2} \qquad \text{(III.2)}$$

$$(\Delta L_1)^2(\Delta L_2)^2 \geq \frac{\hbar^2}{4}\langle L_3\rangle^2{}_{|L2\approx 0\rangle} \approx 0 \Rightarrow \Delta p\Delta q << \frac{\hbar}{2} \qquad \text{(III.3)}$$

IV. Discussion

Both Green and Finkelstein present theories depicting space-time as a contraction of a fundamentally operationally characterized network of units of process or information, on a nanoscale. What are some of the ramifications for a future information theory resulting in potentially feasible technological applications?

Consider Finkelstein's finite harmonic oscillator. In the extreme cases where ordinary quantum field theory fails --in the infrared and ultraviolet cases--the 'hard' and 'soft' oscillators instead "cheat" the Heisenberg uncertainty relations. Hence, similar to the case of squeezed states of EM radiation,[21] manipulations of quanta on that scale in these cases can likewise be hypothetically performed with no theoretical bound to their accuracy. In addition, the energy required for such transactions could be provided by the time-energy conversion at the Planck power scale.

In the case of H. S. Green, his extended qubits comprise the very essence of space-time. Recall that "an informationally based theory could well provide…indication of differences [between photon and neutrino-based physical geometries] which in the future could be detected experimentally." (2000,147) The question then becomes: if a field represents the transmission of information, how accessible is such information at the nanoscale?

In an intriguing application in quantum optics and information theory, Asher Peres and Daniel Terno (2004) seem to substantiate Green's notions, as they work out the effective density matrix for a monochromatic signal consisting of a single photon.[22] For a Fourier decomposition, the Cartesian components of the photon's wave 4-vector k_μ can

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[21] I.e. coherent states $|\alpha,s\rangle$ with an additional degree of freedom (the 'squeezing' parameter s) resulting in violations of Heisenberg Uncertainty.

[22] "When we consider individual photons, for cryptographic applications…quantum theory becomes essential. The diffraction effects…lead to superselection rules which make it impossible to define a reduced density matrix for polarization. [However]…it is still possible to have effective density matrices…[which] depend on the preparation process, [and] also on the method of detection that is used by the observer." (Peres & Terno, 2004, 16)

be expressed in terms of spherical polar angles (θ,ϕ): $k_\mu = (1, \sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$. Then, when considering the effect of the motion of a detector propagating with constant velocity $\boldsymbol{v} = (0,0,v)$ for small θ (when $\theta^2 << |v|$) its Lorentz-transformed component in the detector frame reduces to:

$$\theta' = \theta\sqrt{\frac{1+v}{1-v}} \qquad \text{(IV.1)}$$

This is the same Doppler factor derived by Jarrett & Cover (1981), *absent any specific physical model*, for the relativistic transformation of bit rate and noise intensity. "This remarkable agreement shows that information theory should properly be considered as a branch of physics." (Peres & Terno (2004), 16)

Peres and Terno, in fact, are also motivated by the belief in the inseparability of the disciplines of relativity theory, quantum theory, and information theory. (2004, 3) Contrary to the theories of Green and Finkelstein, however, (proceeding "top down" from abstract mathematical considerations) Peres and Terno work "bottom up" from a POVM[23] formalism modeling *actual experimental* processes of detector emission and absorption. They carry out their results with a fundamentally *algebraic* approach to field theory, as a means of solving some of the difficulties associated with the predictions depending upon specific methods of calculation, when working with different PVMs in curved space-time.[24] Hence, the theoretical entities in Green and Finkelstein's theories

[23] Positive operator valued measure
[24] "One of the difficulties of QFT in curved space-times is the absence of a unique (or preferred) Hilbert space…[since] different representations of canonical commutation or anticommutation relations lead to unitarily inequivalent representations." (Peres & Terno, 2004, p.24).

can be characterized in Peres and Terno, via respective algebraic homomorphisms. This constitutes the first step toward a future information theory of vacuum microtopology.

V. Conclusion

The empirical tests discussed by Green and Finkelstein, vis-à-vis the POVM detector formalism of Peres and Terno raise compelling questions for future information-based technologies. Since the work of Shannon and Hawking in the fifties and sixties, compelling associations among entropy, information, and gravity emerged in the study of Hawking radiation. Nowadays, however, the theories of Green and Finkelstein together suggest that the study of space-time may not end at the edge of a black hole's event horizon, but begin in the development of technologies better able to probe its microtopology in controlled laboratory conditions.

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