Name:______________________________ **Representation of Functions as Power Series**

Group Members:

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1. All of the following series are geometric. Indicate a and r and determine the interval of convergence, then find the sum of each of the series. (Your answer will be in terms of x).

   (a) $\displaystyle\sum_{n=0}^{\infty} 2\left(\frac{x}{3}\right)^n = \frac{2}{1-\frac{x}{3}} = \frac{6}{3-x}$

   $a = 2, r = \dfrac{x}{3}$

   (b) $\displaystyle\sum_{n=0}^{\infty}(3x)^n = \frac{1}{1-3x}$

   $a = 1, r = 3x$

   (c) $\displaystyle\sum_{n=0}^{\infty}(-1)^n x^{2n} = \frac{1}{1+x^2}$

   $a = 1, r = -x^2$

2. Find a power series whose sum is given by the following functions.

   (a) $\displaystyle f(x) = \frac{3}{1+x} = \frac{3}{1-(-x)} = \sum_{n=0}^{\infty} 3(-x)^n = \sum_{n=0}^{\infty} 3(-1)^n x^n$

   (b) $\displaystyle f(x) = \frac{1}{1+x^2} = \frac{1}{1-(-x^2)} = \sum_{n=0}^{\infty}(-x^2)^n = \sum_{n=0}^{\infty}(-1)^n x^{2n}$

   (c) $\displaystyle f(x) = \frac{x}{1+x^2} = x \cdot \frac{1}{1-(-x)^2} = x\sum_{n=0}^{\infty}(-x^2)^n = \sum_{n=0}^{\infty}(-1)^n x^{2n+1}$

   (d) $\displaystyle f(x) = \frac{1}{4+x} = \frac{1\cdot\frac{1}{4}}{(4+x)\cdot\frac{1}{4}} = \frac{\frac{1}{4}}{1+\frac{x}{4}} = \frac{\frac{1}{4}}{1-(-\frac{x}{4})} = \sum_{n=0}^{\infty}\frac{1}{4}\left(-\frac{x}{4}\right)^n = \sum_{n=0}^{\infty}\frac{(-1)^n x^n}{4^{n+1}}$