

Philosophy 60 **Solution**
Homework 5

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Instructions: Prove the following using natural deduction. Type your name.
Type your answers. Staple or paper clip multiple pages. If you turn this in by e-mail, type the date and time you sent it on the hardcopy. Each problem 2 pts.
10 pts total.

1. $\forall x(Fx \rightarrow \exists yFxy), \exists zFz \therefore \exists x\exists yFxy$

1.	$\forall x(Fx \rightarrow \exists yFxy)$	A
2.	$\exists zFz$	A
3.	Show: $\exists x\exists yFxy$	
4.	Fa	$\exists E, 2$
5.	$Fa \rightarrow \exists yFay$	$\forall E, 1$
6.	$\exists y Fay$	$\rightarrow E, 4, 5$
7.	$\exists x\exists yFxy$	$\exists I, 6$

2. $\forall x \forall y (Fxy \rightarrow Fyx), \exists x \exists y \neg Fxy \therefore \exists x \exists y \neg Fyx$

1.	$\forall x \forall y (Fxy \rightarrow Fyx)$	A
2.	$\exists x \exists y \neg Fxy$	A
3.	Show $\exists x \exists y \neg Fyx$	
4.	$\exists y \neg Fya$	$\exists E, 2$
5.	$\neg Fba$	$\exists E, 4$
6.	$Fab \rightarrow Fba$	$\forall E, 1$ (twice)
7.	$\neg Fab$	$\rightarrow E^* 5, 6$
8.	$\exists x \exists y \neg Fyx$	$\exists I, 7$ (twice)

3. $\forall x \neg Fx \therefore \neg \exists x Fx$ (Since this is a proof of one form of QN, do not use QN in your proof. All other rules available.)

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|----|---------------------------|----------------|
| 1. | $\forall x \neg Fx$ | A |
| 2. | Show: $\neg \exists x Fx$ | |
| 3. | $\exists x Fx$ | AIP |
| 4. | Fa | $\exists E, 3$ |
| 5. | $\neg Fa$ | $\forall E, 1$ |

4. $\forall x(Fx \vee Ga) \rightarrow \forall x(Fx \vee \exists yGy)$

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|-----|--|--------------------|
| 1. | Show: $\forall x(Fx \vee Ga) \rightarrow \forall x(Fx \vee \exists yGy)$ | |
| 2. | $\forall x(Fx \vee Ga)$ | ACP |
| 3. | Show $\forall x(Fx \vee \exists yGy)$ | |
| 5. | Show $Fb \vee \exists yGy$ | |
| 6. | $Fb \vee Ga$ | $\forall E, 2$ |
| 7. | Show $Fb \rightarrow (Fb \vee \exists yGy)$ | |
| 8. | Fb | ACP |
| 9. | $Fb \vee \exists yGy$ | $\vee I, 8$ |
| 10. | Show $Ga \rightarrow (Fb \vee \exists yGy)$ | |
| 11. | Ga | ACP |
| 12. | $\exists yGy$ | $\exists I, 11$ |
| 13. | $Fb \vee \exists yGy$ | $\vee I, 12$ |
| 14. | $Fb \vee \exists yGy$ | $\vee E, 6, 7, 10$ |

5. $\neg \exists x(Fx \ \& \ \neg Gx) \therefore \forall x (Fx \rightarrow Gx)$

1.	$\neg \exists x(Fx \ \& \ \neg Gx)$	A
2.	Show: $\forall x (Fx \rightarrow Gx)$	
3.	Show: $Fa \rightarrow Ga$	
4.	Fa	
5.	$\forall x \neg(Fx \ \& \ \neg Gx)$	QN, 1
6.	$\neg(Fa \ \& \ \neg Ga)$	$\forall E$, 5
7.	$\neg Fa \vee \neg \neg Ga$	$\neg \&$, 6
8.	$\neg \neg Ga$	$\vee E^*$, DN 4,7
9.	Ga	$\neg \neg$, 8