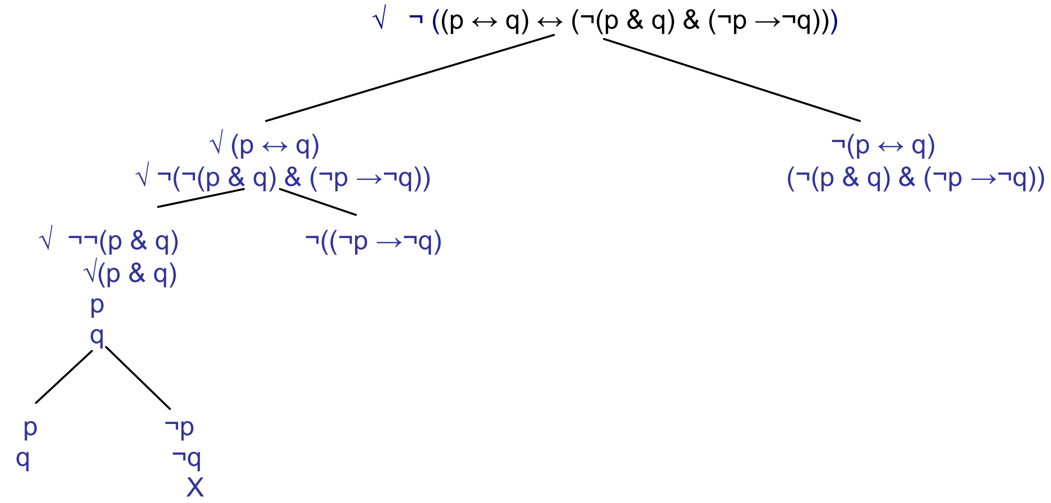
Practice Midterm for Philosophy 60
25 points total

1. Use both a truth table and a truth tree to determine whether the following formula is valid. Explain the results in each case. (5pts.)

(p ↔ q) ↔ (¬(p & q) & (¬p →¬q))

p	q		(p	↔	q)	↔	(¬	(p	&	q)	&	(¬	p	→	¬	q)
T	T		T	T	T	F	F	T	T	T	F	F	T	T	F	T
T	F															
F	T															
F	F															

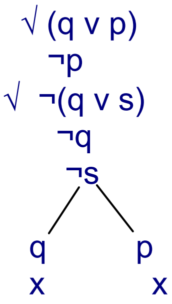
Is not a tatutology if a single interpretation turns out false. No need to finish table.



If the formula is a tautology, every branch of the truth tree of the formula's negation will close. The left hand most branch does not close, therefore this is not a tautology.

2. Use <u>both</u> a truth tree and natural deduction using <u>basic rules</u> only to show that the following argument form is valid. (5pts.)

(q v p); ¬p ∴ q v s



All branches close, hence valid.

1.	(q v p)	A
2.	¬p	A
3.	~~Show~~ (q v s)	
4.	~~Show~~ q → (q v s)	
5.	q	ACP
6.	q v s	vI, 5
7.	~~Show~~ p → (qvs)	
8.	p	ACP
9.	~~Show~~ (q v s)	
10.	¬(q v s)	AIP
11.	p	R, 8
12.	¬p	R, 2
13.	(q v s)	vE, 1,4,7

3. Prove the following argument form is valid by natural deduction using <u>basic rules</u> only. (5pts.)

p ↔ (q v r); ¬p ↔ (q & r) ∴ ¬(q & r)

1.	p ↔ (q v r)	A
2.	¬p ↔ (q & r)	A
3.	~~Show~~ ¬(q & r)	
4.	q & r	AIP
5.	q	
6.	q v r	vI, 5
7.	p	↔E, 1,6
8.	¬p	↔E, 2,4

4. Prove the following argument form is valid using natural deduction. (5 pts.)

(p → (q v r)); (s → (q v t)); (p v s); (¬ r & ¬ t) ∴ q

1.	(p → (q v r))	A
2.	(s → (q v t))	A
3.	(p v s)	A
4.	(¬ r & ¬ t)	A
5.	~~Show q~~	
6.	¬q	AIP
7.	¬r	&E, 4
8.	¬q & ¬r	&I, 6,7
9.	¬(q v r)	¬v, 8
10.	¬p	1,9 →E*
11.	s	3, 10 vE*
12.	q v t	→E, 11,2
13.	t	vE* 6,12
14.	¬t	&E, 4

5. Prove that the following formula is valid using natural deduction. (5 pts.)

((p ↔ (r → s)) & (q ↔ (s →r))) → ((p & q) ↔ (r ↔ s)))

1.	~~Show~~: ((p ↔ (r → s)) & (q ↔ (s →r))) → ((p & q) ↔ (r ↔ s)))	
2.	(p ↔ (r → s)) & (q ↔ (s →r))	ACP
3.	(p ↔ (r → s))	&E, 2
4.	(q ↔ (s →r))	&E, 2
5.	~~Show~~: (p & q) → (r ↔ s))	
6	(p & q)	ACP
7.	p	&E, 6
8.	q	&E, 6
9.	(r → s)	↔E, 7, 3
10.	(s → r)	↔E, 8, 4
11.	(r ↔s)	↔I, 9,10
12.	~~Show~~ (r ↔s) → (p & q)	
13.	(r ↔ s)	ACP
14.	(r → s) & (s → r)	↔ E, 13
15.	(r → s)	&E, 14
16	(s → r)	&E, 14
17.	p	↔E, 3,15
18.	q	↔E, 4,16
19.	(p & q)	&I, 17,18
20.	((p & q) ↔ (r ↔ s))	↔I, 5,12

Note that in this proof it turned out to be a really good idea to do the &E on 3,4 before starting the conditional proofs on lines 6 and 12. If we hadn't done that, we would have performed this operation twice, internal to both conditional proofs.