

Practice for Test 6

The solutions to these proofs are on the following pages. I strongly recommend that you do not look at these solutions until you are either confident that you have solved the problems on your own or you are completely stymied. If the latter, you should only look at as much of the solution as you need to get started. If you think you detect an error in one of the solutions, please let me know. The actual test will contain only two problems.

Problem 1.

1. $s \rightarrow (p \leftrightarrow q)$ A
2. $r \rightarrow p$ A
3. $p \rightarrow q$ A
4. $s \vee r$ A
5. Show $(p \leftrightarrow q) \vee (p \wedge q)$

Problem 2

1. $(t \rightarrow (p \leftrightarrow q))$ A
2. $(k \rightarrow (r \leftrightarrow s))$ A
3. $(t \wedge k)$ A
4. Show: $((p \wedge s) \rightarrow (q \vee r)) \wedge ((q \wedge r) \rightarrow (p \vee s))$

Problem 3

1. $(p \wedge q) \vee (p \rightarrow q)$ A
2. $(p \vee t) \rightarrow r$ A
3. $(q \vee s) \rightarrow r$ A
4. $(p \rightarrow q) \rightarrow (\neg r \rightarrow \neg s)$ A
5. s A
6. Show: $r \vee s$

Problem 1.

1.	$s \rightarrow (p \leftrightarrow q)$	A
2.	$r \rightarrow p$	A
3.	$p \rightarrow q$	A
4.	$s \vee r$	A
5.	Show $(p \leftrightarrow q) \vee (p \& q)$	
6.	Show $s \rightarrow ((p \leftrightarrow q) \vee (p \& q))$	
7.	s	ACP
8.	$(p \leftrightarrow q)$	$\rightarrow E, 1, 7$
9.	$((p \leftrightarrow q) \vee (p \& q))$	$\vee I, 8$
10.	Show $r \rightarrow ((p \leftrightarrow q) \vee (p \& q))$	
11.	r	ACP
12.	p	$\rightarrow E, 2, 11$
13.	q	$\rightarrow E, 3, 12$
14.	$p \& q$	$\& I, 12, 13$
15.	$((p \leftrightarrow q) \vee (p \& q))$	$\vee I, 14$
16.	$((p \leftrightarrow q) \vee (p \& q))$	$\vee E, 4, 6, 10$

Problem 2

1.	$(t \rightarrow (p \leftrightarrow q))$	A
2.	$(k \rightarrow (r \leftrightarrow s))$	A
3.	$(t \& k)$	A
4.	Show $((p \& s) \rightarrow (q \vee r)) \& ((q \& r) \rightarrow (p \vee s))$	
5.	Show $(p \& s) \rightarrow (q \vee r)$	
6.	$p \& s$	ACP
7.	t	$\& E, 3$
8.	$p \leftrightarrow q$	$\rightarrow E, 1, 7$
9.	p	$\& E, 6$
10.	q	$\leftrightarrow E, 8, 9$
11.	$q \vee r$	$\vee I, 10$
12.	Show $(q \& r) \rightarrow (p \vee s)$	
13.	$(q \& r)$	ACP
14.	k	$\& E, 3$
15.	$r \leftrightarrow s$	$\rightarrow E, 2, 14$
16.	r	$\& E, 13$
17.	s	$\leftrightarrow E, 15, 16$
18.	$p \vee s$	$\vee I, 17$
19.	$((p \& s) \rightarrow (q \vee r)) \& ((q \& r) \rightarrow (p \vee s))$	$\& I, 5, 12$

Problem 3

1.	$(p \& q) \vee (p \rightarrow q)$	A
2.	$(p \vee t) \rightarrow r$	A
3.	$(q \vee s) \rightarrow r$	A
4.	$(p \rightarrow q) \rightarrow (\neg r \rightarrow \neg s)$	A
5.	s	A
6.	Show: $r \vee s$	
7.	Show $(p \& q) \rightarrow (r \vee s)$	
8.	$p \& q$	ACP
9.	p	
10.	$p \vee t$	$\vee I, 9$
11.	r	$\rightarrow E, 2, 10$
12.	$r \vee s$	$\vee I, 11$
13.	Show $(p \rightarrow q) \rightarrow (r \vee s)$	
14.	$p \rightarrow q$	ACP
15.	Show $\neg \neg r$	
16.	$\neg r$	AIP
17.	$\neg r \rightarrow \neg s$	$\rightarrow E, 4, 14$
18.	$\neg s$	$\rightarrow E, 16, 17$
19.	s	R, 5
20.	r	$\neg \neg, 15$
21.	$r \vee s$	$\vee I, 20$
22.	$r \vee s$	$\vee E, 1, 7, 13$