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Chapter 1 Basic Concepts of Logic

Logic defined

Logic is the study of correct reasoning.

- Informal logic is the attempt to represent correct reasoning using the natural language (English, Spanish, Chinese, etc.) in which the reasoning occurs.
- Formal logic is the attempt to represent correct reasoning in a symbolic language that is created for the specific purpose of representing logical relationships.

Analogy with mathematics

- An analogy with mathematics can help to clarify the difference. You can write:
 - Three times seven equals twenty-one.
- And you can write:

□ 3 x 7 = 21

They both mean the same thing, but the first sentence is written in English and the second is written in mathematics. Obviously, the reason you will usually prefer to write the expression in the second way is that it represents mathematical relationships more precisely, and makes mathematical calculations a lot easier.

Arguments

- In logic we represent reasoning with structures called <u>arguments</u>.
- An argument is just a series of statements, called <u>premises</u>, that are meant to support another statement called the <u>conclusion</u>.
- It's important to be aware that in logic the term "argument" doesn't have anything to do with the ordinary sense of arguments as a heated verbal exchange. It's true that people may engage in ordinary arguments by making arguments in our technical sense of the term, but people who just shout their opinions at each other giving no rational basis for them are also ordinarily said to be arguing.

Informal vs. formal argument.

- Here is an argument in English:
 - My assignment would be better if I took one extra day to complete it. But if I took one extra day to complete it the assignment would be downgraded by 20%. I don't think I can make it more than 10% better in one day, so I might as well just turn it in now.
- This argument can be put into what logicians call "standard form" as follows.
 - 1. If I take one extra day to complete my assignment, it will be better.
 - 2. If I take one extra day to complete the assignment it will be downgraded by 10%.
 - 3. I can't make the assignment more than 20% better in one day.
 - 4. Therefore, I should turn the assignment in today.
- that standard form is still English. It simply involves making a list of the premises with the conclusion at the end. We'll soon learn a formal language called "sentential logic". If we were to use sentential logic to represent the argument above it would look something like this:
- 1. $p \rightarrow q$ 2. $p \rightarrow r$ 3. S 4. t

Correct reasoning

- We said that logic is the study of correct reasoning. In logic, correct reasoning is reasoning that preserves truth. Put differently, correcting reasoning is reasoning in which true premises leads to true conclusions.
- When we put this technically, we use the term "validity" or more precisely still "deductive validity". The precise definition of deductive validity is this:
 - An argument is deductively valid if and only if its conclusion is true whenever its premises are all true. (p.17).
- Again, the term "valid" is a technical term for us. In English we use the term to mean a lot of different things, but in logic it means just what we said and nothing else. There are ways of expressing logical validity that mean the same thing, however. Here are two other common and equivalent ways of defining validity.
 - An argument is deductively valid if and only if it is impossible for the premises to be true and the conclusion false.
 - An argument is deductively valid if and only if, given the truth of the premises, the conclusion must also be true.

Validity vs. truth

- It is very common for people to <u>think</u> they understand the concept of validity when they are actually confusing it with the concept of truth. For example, even though they can define the concept of deductive validity, many people will say that the following argument is not valid.
 - 1. All dogs can whistle.
 - 2. All cats can yodel.
 - 3. Everything is either a cat or a dog.
 - 4. Therefore, everything can either whistle or yodel.
- People who say the argument is not valid are usually just temporarily forgetting what validity means. It's pretty obvious that whenever the first three statements are true, the conclusion is true. The problem is that the first three statements are so clearly false that we find ourselves wanting to call it invalid for that reason.
- So the important point to remember is that validity has nothing to do with the actual truth of the premises. It only has to do with what happens to the truth of the conclusion if we <u>assume</u> the premises are true.

Why we need formal logic

- Just like doing complicated math problems in English, demonstrating whether an argument made in English is deductively valid can be extremely difficult.
- For example, is this argument valid?
 - Bob loves only people who don't love him. So if Bob loves himself, then Bob is a vampire.
- It is certainly bizarre, but the words "bizarre" and "invalid" are not synonyms. It turns out that this argument actually is deductively valid. But we need formal logic to demonstrate this.
- If you like the mathematical analogy, needing formal logic to demonstrate the validity of this argument is like needing mathematics to show that you really can subtract something from nothing. It doesn't seem to make intuitive sense, but it actually makes perfect mathematical sense.

Logical properties of sentences.

- Arguments are sets of sentences that bear a logical relationship to each other, but sentences themselves also have logical properties. Like validity, these properties are also defined in terms of truth.
- All of the following properties express various relationships that sentences may have to truth.

Contingent sentences

- A sentence is <u>contingent</u> if and only if it is possible for it to be true and possible for it to be false. (p. 27)
- This is by far the most common kind of statement we make, but it can still be easy to get confused about the nature of contingency. The important thing to understand is that <u>possibility</u> is itself a logical notion.
- This statement is obviously contingent
 - It's raining in Sacramento.
 - because on any given day it's possible that it's raining and possible that it's not. But thesestatement is also contingent
 - Mahmoud Ahmadinejad is president of the United States.
 - Bob turned into a giant carrot.
- The first statement is patently false and the second is seemingly physically impossible (though it isn't, really), but they are both logically possible. There is, in other words, some possible state of affairs in which Ahmadinejad becomes the president of the U.S.. And there is a possible reorganization of the matter that currently constitutes Bob into an arrangement that would constitute a giant carrot.

Valid and contradictory sentences.

- Sentences that aren't contingent are either <u>valid</u> or <u>contradictory</u>.
- A sentence is <u>valid</u> (tautologous, or logically true) if and only if it is true in every possible circumstance. (p.27)
 - Enough is enough.
 - Whatever will be, will be.
- A sentence is <u>contradictory</u> if and only if it is impossible for it to be true. (p. 28)
 - Barry hit a five run homer.
 - Neither of Marcie's grandmothers had children.

Satisfiability

- Finally, a sentence is said to be <u>satisfiable</u> if and only if is not contradictory (p. 29).
- Another way to say this is that a sentence is satisifiable if and only if it is either valid or contingent.
- We also speak of <u>sets</u> of sentences as being satisfiable or contradictory.
 - A set of sentences is contradictory if and only if it is impossible for all of the sentence to be true.
 - □ A set of sentences is satisfiable if it is not contradictory.

Satisfiability and Validity

- There is an interesting connection between the concepts of deductive validity, satisfiability, and contradiction.
- Consider an obviously deductively valid argument like:
 - 1. Some worms are vicious.
 - 2. All vicious things are delicious.
 - 3. So, some worms are delicious.
- Now, suppose you deny the conclusion of this argument and rewrite it simply as a set of sentences.
 - 1. Some worms are vicious.
 - 2. All vicious things are delicious.
 - 3. No worms are delicious.
- This set of sentences is not satisifiable. In other words, it is impossible for all of these sentences to be false. In other words, it is contradictory.
- This gives us another way to understand validity. A valid argument is one in which denying the conclusion creates a contradictory set of sentences.