Rules for natural deduction

Assumption (A)

n. $\mathcal{A}$ A

This rule must be applied before the first show line.

Reiteration (R)

n. $\underline{\mathcal{A}}$
n+p. $\mathcal{A}$ R, n

Conjunction Exploitation (&E)

n. $\underline{\mathcal{A} \,\&\, \mathcal{B}}$
n+m $\mathcal{A}$ (or $\mathcal{B}$) &E, n

Conjunction Introduction (&I)

n. $\mathcal{A}$
m. $\underline{\mathcal{B}}$
p. $\mathcal{A} \,\&\, \mathcal{B}$ &I, n, m

Negation Introduction/Exploitation

n. $\mathcal{A}$
========
m. $\neg\neg\mathcal{A}$ $\qquad \neg\neg$, n

Conditional Exploitation

n. $\mathcal{A} \rightarrow \mathcal{B}$
m. $\underline{\mathcal{A}}$
$\quad \mathcal{B}$ $\qquad \rightarrow$E, n,m

Biconditional Introduction

n. $\mathcal{A} \rightarrow \mathcal{B}$
m. $\underline{\mathcal{B} \rightarrow \mathcal{A}}$
p. $\mathcal{A} \leftrightarrow \mathcal{B}$ $\qquad \leftrightarrow$I, n, m

Biconditional Exploitation

n. $\mathcal{A} \leftrightarrow \mathcal{B}$
m. $\underline{\mathcal{A}\ \ (\text{or}\,\mathcal{B})}$
$\quad \mathcal{B}\ \ (\text{or}\,\mathcal{A})$ $\qquad \leftrightarrow$E, n, m

Disjunction Introduction

n. $\underline{\mathcal{A}}$
m. $\mathcal{A} \vee \mathcal{B}$ vI, n

Disjunction Exploitation

n. $\mathcal{A} \vee \mathcal{B}$
m. $\mathcal{A} \rightarrow \mathcal{C}$
p. $\underline{\mathcal{B} \rightarrow \mathcal{C}}$
q. $\mathcal{C}$ vE, n, m, p

<u>Proof Rules</u>

Direct Proof (page 109)

(p v r) ↔ q; ¬¬p ∴ q

1.	(p v r) ↔ q	A
2.	¬¬p	A
3.	~~Show~~ q	
4.	p	¬¬, 2
5.	p v r	vI, 4
6.	q	↔E, 1,5

Conditional Proof (page 119)

$p \vee (q \,\&\, r) \;\therefore\; \neg q \rightarrow p$

1.	$p \vee (q \,\&\, r)$	A
2.	~~Show~~ $\neg q \rightarrow p$	
3.	$\neg q$	ACP
4.	$\neg q \vee \neg r$	vI, 3
5.	$\neg(q \,\&\, r)$	¬&, 4
6.	p	vE* 1,5

Indirect Proof (page 126, liberalized version)

$q \leftrightarrow (p \vee r);\; \neg p;\; \neg r \;\therefore\; \neg q$

1.	$q \leftrightarrow (p \vee r)$	A
2.	$\neg p$	A
3.	$\neg r$	A
4.	~~Show~~ $\neg q$	
5.	q	AIP
6.	$p \vee r$	↔E, 1,5
7.	$\neg p \,\&\, \neg r$	&I, 2,3
8.	$\neg(p \vee r)$	¬v 7
9	$p \vee r$	R, 6

Table of Basic and Derived Rules

The symbol ├ means "yields" or "implies". When there are two of them, written like this ┤├ it means the formulas yield each other, or are logically equivalent. Bonevac does not use this symbol, but it's convenient for representing the rules in a linear fashion.

Rule	**Symbol**
Implications	
(A & B) ├ A (A & B) ├ B	&E
A ├ (A v B)	vI
A, (A→B) ├ B	→E
¬B, (A→B) ├ ¬A	→E*
(A v B), (A→C), (B→C) ├ C	vE
¬A, (A v B) ├ B ¬B, (A v B) ├ A	vE*
A, B ├ (A & B)	&I
(A→B), (B→C) ├ (A→C)	→→
(A→B), (B→A) ├ (A↔B)	↔I
(A ↔B), A ├ B (A ↔B), B ├ A	↔E
(A ↔B), ¬B ├ ¬A (A ↔B), ¬A ├ ¬B	↔E*
(A↔B) ├ (A→B) & (B→A)	↔→
A, ¬A ├ B	!
Equivalences	
¬¬A ┤├ A	¬¬
¬ (A v B) ┤├ (¬A & ¬B) ¬ (A & B) ┤├ (¬A v ¬B)	¬v ¬&
(A & (B & C)) ┤├ ((A & B) & C) (A v (B v C)) ┤├ ((A v B) v C)	&A vA
(A & B) ┤├ (B & A) (A v B) ┤├ (B v A)	&C vC
¬ (A→B) ┤├ (A & ¬B)	¬→
¬ (A↔B) ┤├ (A ↔ ¬B) ¬ (A↔B) ┤├ (¬A ↔ B)	¬↔
(A→B) ┤├ (¬A v B)	→v