Midterm for Philosophy 60
25 points total

1. Use a truth table to determine whether the following formula is a contradiction. Explain the results. (5pts.)

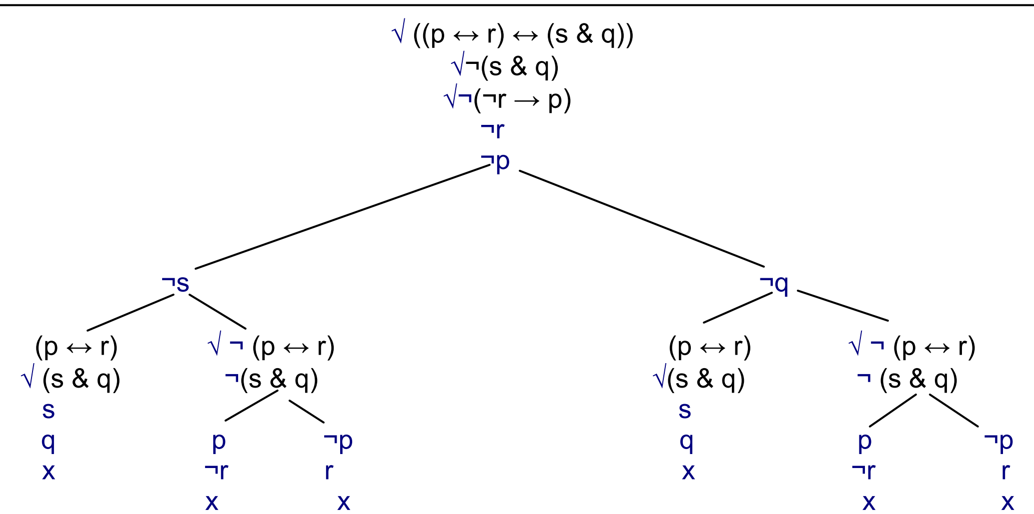
((p ↔ q) ↔ (¬(p & q) & (p v q)))

p	q		((p	↔	q)	↔	(¬	(p	&	q)	&	(p	v	q)))		
T	T		T	T	T	F	F	T	T	T	F	T	T	T		
T	F		T	F	F	F	T	T	F	F	T	T	T	F		
F	T		F	F	T	F	T	F	F	T	T	F	T	T		
F	F		F	T	F	F	T	F	F	F	F	F	F	F		

All False under main (blue column) connective, hence contradiction.

2. Use a truth tree to determine whether the following argument form is valid. (5pts.)

((p ↔ r) ↔ (s & q)); ¬(s & q) ∴ (¬r → p)



All branches close, therefore valid.

3. Prove the following argument form is valid by natural deduction using <u>basic rules</u> only. (5pts.)

(p v (q v r)); (r → s);(p → s); ¬q ∴ s

1,	(p v (q v r))	A
2.	(r → s)	A
3.	(p → s)	A
4.	¬q	A
5.	~~Show s~~	
6.	~~Show~~ (q v r) → s	
7.	(q v r)	ACP
8.	~~Show~~ (q → s)	
9.	q	ACP
10.	~~Show~~ s	
11.	~~Show ¬¬s~~	
12.	¬s	AIP
13.	q	R, 4
14.	¬q	R, 9
15.	s	¬¬11
16.	s	VE, 7,2,8
17.	s	VE, 1,3,6

4. Prove the following argument form is valid using natural deduction. (5 pts.)

¬(p → q); (p → (s v r)); (¬q → (¬s & ¬r)) ∴ (p ↔ m)

1	¬(p → q)	A
2.	(p → (s v r))	A
3.	(¬q → (¬s & ¬r))	A
4.	~~Show~~ (p ↔ m)	A
5.	p & ¬q	¬→,1
6.	p	
7.	¬q	
6.	s v r	→E, 2,6
7.	¬s & ¬r	→E, 3,7
8.	¬ r	&E, 7
9.	s	vE*, 6,8
10.	¬s	&E, 7
11	(p ↔m)	!9, 10

5. Prove that the following formula is valid using natural deduction. (5 pts.)
(¬(p v (q & s)) ↔ (¬(¬p → q) v ¬(¬p → s))

1.	~~Show~~: (¬(p v (q & s)) ↔ (¬(¬p → q) v ¬(¬p → s))	
2.	~~Show~~ (¬(p v (q & s)) → (¬(¬p → q) v ¬(¬p → s))	
3.	¬(p v (q & s))	ACP
4.	¬p & ¬(q & s)	¬v, 2
5.	¬p	&E, 3
6.	¬(q & s)	&E, 3
7.	~~Show~~ (¬(¬p → q) v ¬(¬p → s)	
8.	¬ (¬(¬p → q) v ¬(¬p → s)	AIP
9.	¬¬(¬p → q) & ¬¬(¬p → s)	¬v,7
10	(¬p → q) & (¬p → s)	¬¬ (twice)
11	(¬p → q)	&E, 10
12	(¬p → s)	&E, 10
13	q	→E, 4,11
14	s	→E, 4, 12
15	(q & s)	&I, 13, 14
16	¬(q & s)	R, 5
17	~~Show~~: ((¬(¬p → q) v ¬(¬p → s)) → ¬(p v (q & s)))	
18	(¬(¬p → q) v ¬(¬p → s))	ACP
19	~~Show~~ ¬(¬p → q) → ¬(p v (q & s))	
20	¬(¬p → q)	ACP
21	¬p & ¬q	¬→, 20
22	¬p	&E, 21
23	¬q	&E, 22
24	¬q v ¬s	v I, 23
25	¬(q & s)	¬&, 24
26	¬p & ¬(q & s)	&I, 22,25
27	¬(p v (q & s))	¬v, 26
28	~~Show~~: ¬(¬p → s) → ¬(p v (q & s))	
29	¬(¬p → s)	ACP
30	¬p & ¬s	¬→, 29
31	¬p	&E, 30
32	¬s	&E, 30
33	¬s v ¬q	vI, 32
34	¬q v ¬s	vC, 33
35	¬(q & s)	¬&, 34
36	(¬p & ¬(q & s))	&I, 31,35
37	¬(p v (q & s))	¬v, 36
38	¬(p v (q & s))	vE, 18, 19, 28
39	(¬(p v (q & s)) ↔ (¬(¬p → q) v ¬(¬p → s))	↔I, 2, 17

Extra Credit 2pts.

Determine whether the following argument is valid in sentential logic by any approved method. What does the answer to this question tell you about the truth-functionality of the connective "if…then?".

It is not the case that if Simon loses his virginity, then he will become pregnant. Therefore, Simon will lose his virginity and he will become pregnant.

$\surd\ \neg(s \rightarrow p)$
$\neg(s \,\&\, p)$
s
$\neg p$

$\neg s$ $\neg p$
x

Not valid because one branch remains open. This is an intuitive result, so it seems that the truth-functional interpretation of "if….then" is satisfactory in this case.

Interestingly, however, the following reasoning is actually valid.

It is not the case that if Simon loses his virginity, then he will become pregnant. Therefore, Simon will lose his virginity and he will not become pregnant.

$\surd\ \neg(s \rightarrow p)$
$\neg(s \,\&\, \neg p)$
s
$\neg p$

$\neg s$ $\neg\neg p$
x   x

This is an unintuitive result and suggests that the truth-functional interpretation of "if…then" may not be appropriate in this case.