

Is there a difference in meaning between the phrases “One in a Hundred” and “A Hundred to one Shot”?

The first phrase should be interpreted as a probability  $1/100$ . That is, out of every 100 attempts, there should be 1 occurrence. However, the second phrase is stated as a ratio  $100 : 1$ , and should therefore be interpreted as odds. We formally define “odds in favor” and “odds against” as follows:

### Fraction Version

Let  $p = \frac{a}{n}$  be the probability that event  $A$  occurs, and let  $q = \frac{b}{n}$  be the probability that  $A$  does not occur, where  $a + b = n$

- (i) If  $p \geq \frac{1}{2}$ , then we define the *odds in favor of A* to be the ratio  $a : b$  (which should be simplified algebraically).
- (ii) If  $p < \frac{1}{2}$ , then we define the *odds against A* to be the ratio  $b : a$  (which should be simplified algebraically).

**Example 1.** What are the appropriate odds of drawing a Club from a deck of cards?

*Solution.* The probability of drawing a Club is  $p = 1/4$  which is less than 0.50, so we should state the odds *against* drawing a Club. Since  $p = 1/4$ , then  $q = 3/4$ . Therefore, the odds of drawing a Club are **3 : 1 against**.

(Because  $p = 1/4$ , then for every 4 attempts there should be 1 favorable occurrence and 3 non-favorable; hence, the odds are 3 : 1 *against*.)

**Example 2.** What are the odds of drawing from Two through 10 from a standard deck of 52 cards (excluding Jokers)?

*Solution.* There are 9 desirable values from 13 possible, so  $p = 9/13$  is the probability of success and  $q = 4/13$  is the probability of failure. Thus, the odds are **9 : 4 in favor**.

### Decimal Version

Let  $p = P(A)$  be the probability that event  $A$  occurs, and let  $q = 1 - p$  be the probability that  $A$  does not occur.

- (i) If  $p \geq 0.50$ , then we define the *odds in favor of A* to be the ratio  $p : q$  (which should be simplified algebraically).
- (ii) If  $p < 0.50$ , then we define the *odds against A* to be the ratio  $q : p$  (which should be simplified algebraically).

**Example 3.** In the state, 72% of registered voters are Democrats. If a registered voter is chosen at random, what are the odds of choosing a Democrat?

*Solution.* Because  $p = 0.72$  which is more than 0.50, we should state the odds in favor of choosing a Democrat. So the odds are  $p : q = 0.72 : 0.28 = 72 : 28 = \mathbf{18 : 7}$  in favor.

**Example 4.** Find the appropriate odds if

$$(i) P(A) = \frac{5}{7} \quad (ii) P(A) = 0.32 \quad (iii) P(A) = \frac{2}{13}.$$

(i) Out of every 7 attempts there should be 5 favorable occurrences and 2 non-favorable; hence, the odds are **5 : 2 in favor**.

(ii) Here  $p = 0.32$ , thus  $q = 1 - 0.32 = 0.68$ . Because the chance  $q$  of *not* happening is larger, the odds are  $q : p = 0.68 : 0.32 = 68 : 32 = \mathbf{17 : 8}$  against  $A$  happening.

(iii) Out of every 13 attempts there should be 2 favorable occurrences and 11 non-favorable; hence, the odds are **11 : 2 against**.

### Converting Odds to Probabilities

The odds are always stated as a simplified ratio  $a:b$ , where  $a$  and  $b$  are positive integers and  $a \geq b$ . (The larger number comes first.) Think of the sum  $a+b$  as the total number of possibilities.

If  $a : b$  are the odds *in favor*, then  $a$  is the number of favorable outcomes and  $b$  is the number of non-favorable. Then  $P(A) = \frac{a}{a+b}$ .

If  $c : d$  are the odds *against*, then the number  $c$  coming first is the number of *non-favorable* outcomes. The second number  $d$  is the number of favorable outcomes. Thus  $P(A) = \frac{d}{c+d}$ .

**Example 5.** The odds of event  $A$  are 13 : 3 in favor. What is  $P(A)$ ?

*Solution.* There are  $(13 + 3) = 16$  possibilities, of which 13 are favorable and 3 are non-favorable, so  $P(A) = \frac{13}{13+3} = \frac{13}{16}$ .

**Example 6.** The odds of event  $A$  are 15 : 9 against. What is  $P(A)$ ?

*Solution.* Out of every  $(15 + 9) = 24$  attempts, 15 are *non-favorable* and 9 are favorable, so  $P(A) = \frac{9}{15+9} = \frac{9}{24}$ .

Math 1: Section: \_\_\_\_\_ Date: \_\_\_\_\_ Name: \_\_\_\_\_

**PROBABILITY AND ODDS WORKSHEET**

**Example:** Jessica has a normal deck of cards, which contains 52 cards and 13 cards are hearts. She asks her friend Sarah to draw a card.

What is the probability that Sarah will select the hearts? \_\_\_\_\_

What is the probability that Sarah will not select the hearts? \_\_\_\_\_

What are the odds in favor of Sarah selecting the hearts? \_\_\_\_\_

What are the odds against Sara selecting the hearts? \_\_\_\_\_

**Susie has a spinner that has four sectors. One is yellow, one is orange, one is blue and one is red. Calculate the following:**

1. What is the probability of spinning a yellow? \_\_\_\_\_

2. What is the probability of not spinning a yellow? \_\_\_\_\_

3. What are the odds in favor of spinning a yellow? \_\_\_\_\_

4. What are the odds against spinning a yellow? \_\_\_\_\_

**Sammy has a number cube (dice). Calculate the following:**

5. What is the probability of rolling a 6? \_\_\_\_\_

6. What is the probability of not rolling a 6? \_\_\_\_\_

7. What are the odds in favor of rolling a 6? \_\_\_\_\_

8. What are the odds against rolling a 6? \_\_\_\_\_

9. What is the probability of rolling an even number? \_\_\_\_\_

10. What is the probability of not rolling an even number? \_\_\_\_\_

11. What are the odds in favor of rolling an even number?

\_\_\_\_\_

12. What are the odds against rolling an even number?  
number?

\_\_\_\_\_

**A sweepstakes has 500 entries. You have purchased one ticket. Calculate the following:**

13. What is the probability that you will win the  
sweepstakes?

\_\_\_\_\_

14. What is the probability that you will not win  
the sweepstakes?

\_\_\_\_\_

15. What are the odds in favor of you winning the  
sweepstakes?

\_\_\_\_\_

16. What are the odds against you winning the  
sweepstakes?

\_\_\_\_\_

**Challenge questions:**

17. If you purchase two tickets in the sweepstakes, does that double the probability that you will win?

18. If you purchase two tickets instead of one, use one of the following words to describe the likelihood of you winning the sweepstakes:

*Certain, Impossible, Unlikely*

## Odds against or in favor

### Practice:

1) In her wallet, Anne Kelly has 14 bills. Seven are \$1 bills, two are \$5 bills, four are \$10 bills and one is a \$20 bill. She passes a volunteer seeking donations for the Salvation Army and decides to select one bill at random from her wallet and give it to the Salvation Army.

Determine:

- The probability she selects a \$5 bill
- The probability she does not select a \$5 bill
- The odds in favor of her selecting a \$5 bill
- The odds against her selecting a \$5 bill

2) A box contains 9 red and 2 blue marbles and 3 yellow marbles. If you select one at random from the box, determine:

- The probability the marble is red.
- The odds in favor of selecting a red marble.
- The probability the marble is blue.
- The odds against selecting a blue marble

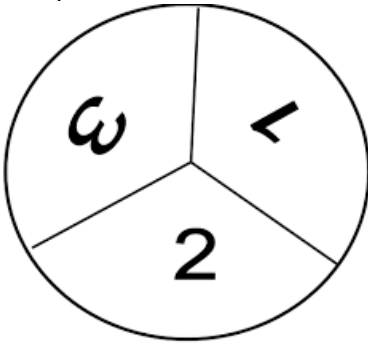
3) A pair of dice is rolled and the sum of the dice is recorded. Here is the sample space.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

- Find the probability of rolling a sum of 7.
- Find the probability of not rolling a sum of 7
- Find the odds in favor of the sum being 7.
- Find the odds against the first dice showing a 5.
- Find the probability the sum is less than 7.
- Find the odds against of the sum being less than 7.
- Find the probability of rolling a double (both dice have the same number).
- Find the odds against rolling a double.

- 4) Use the sample space from problem 15 to answer the following.
- Find the probability of rolling a sum of 8.
  - Find the probability of the first dice shows the number 2.
  - Find the odds in favor of the sum being 8.
  - Find the odds against the first dice showing a 2.
  - Find the probability the sum is greater than 9.
  - Find the odds in favor of the sum being greater than 9.
  - Find the probability of **not** rolling a double (both dice have the same number).
  - Find the odds in favor of the sum being an odd number.
- 5) One person is selected at random from a class of 16 men and 14 women. Find the odds against selecting:
- A woman
  - A man

18 – 21: A dart is thrown at this target and the numbers it lands on is noted. Find the requested odds.



- Odds in favor of landing in the one region.
- Odds in favor of landing in the odd region.
- Odds against landing in the one region.
- Odds against landing in the odd region.

#3 – 14: A card is picked from a deck of cards. Find the odds against and odds in favor of selecting:

- |                          |                         |                         |
|--------------------------|-------------------------|-------------------------|
| 3) a heart               | 4) a red card           | 5) a seven              |
| 6) a queen               | 7) a seven or a queen   | 8) a diamond or a spade |
| 9) a seven and a heart   | 10) a queen and a spade | 11) the three of spades |
| 12) the four of diamonds | 13) a red king          | 14) a black four        |