Fibonacci Sequence & Golden Ratio

Fibonacci was an Italian mathematician from Pisa, who invented the following sequence in the year 1202:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55 ...

After the first two terms, each term is formed by **adding the two previous terms**. Fibonacci used 1 and 1 as his starting numbers, but any two numbers can be used

So here,

2 = 1 + 1

3 = 2 + 1

5 = 3 + 2

8 = 5 + 3

13 = 8 + 5 etc....

Example

Work out the missing term in this Fibonacci sequence:

2, 2, 4, ?, 10, 16 ?...

Answer

Look at the first two terms, which add to give 4, the third term. The missing term is therefore 2 + 4 = **6**

Check that the next terms works too 4 + 6 = 10 and 6 + 10 = 16.

Notation for terms in Fibonacci sequence.

$F_0 = 0$

$F_1 = 1$

$F_2 = F_0 + F_1 = 0 + 1 = 1$

$F_3 = F_1 + F_2 = 1 + 1 = 2$

$F_4 = F_2 + F_3 = 1 + 2 = 3$

$F_5 = F_3 + F_4 = 2 + 3 = 5$

$F_6 = F_4 + F_5 = 3 + 5 = 8$

$F_7 = F_5 + F_6 = 5 + 8 = 13$

Question 1) How to find the third term out of 3 terms in Fibonacci sequence, if we know the last two.

Answer. Just add the last two.

Example: If $F_{12}=144$ and $F_{13}=233$, find $F_{15}=?$

First we need to find $F_{14}=F_{12}+F_{13}=144+233=377$, then $F_{15}=F_{13}+F_{14}=233+377=610$.

Practice: $F_{20}=6765$ and $F_{21}=10946$, find $F_{24}=?$ **Answer is 46368**

Question 2) How to find the first term out of 3 terms in Fibonacci sequence, if we know the last two terms.

Answer. Just subtract the smaller from the larger.

Example: If $F_{12}=144$ and $F_{13}=233$, find $F_{11}=?$

$F_{11}=F_{13}-F_{12}=233-144=89$, in the same fashion, we can find $F_{10}=F_{12}-F_{11}=144-89=55$

then $F_{15}=F_{13}+F_{14}=233+377=610$.

Practice: $F_{20}=6765$ and $F_{21}=10946$, find $F_{19}=?$ and $F_{18}=?$ **Answers:** $F_{19}=4181$ **and** $F_{18}=4181$

Golden ratio, also known as the divine proportion, golden mean, or golden section, is a number often encountered when taking the ratios of two consecutive terms in Fibonacci sequence.

Notation for Golden ratio is the Greek alphabet φ (pronounced fee).

How to **approximate** φ by dividing each term by its previous one.

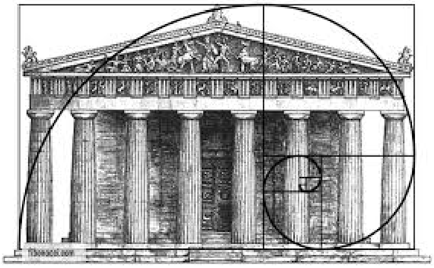
Example: $\varphi=\frac{F_{10}}{F_9}=\frac{55}{34}=1.617647$ or $\varphi=\frac{F_{27}}{F_{26}}=\frac{196418}{121393}=1.618033$

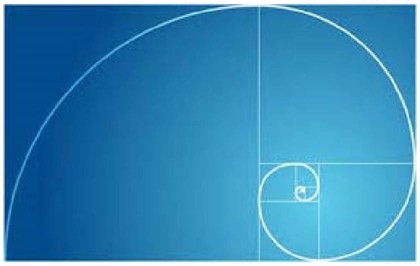
Find $\varphi=\frac{F_7}{F_6}=\frac{?}{?}=$

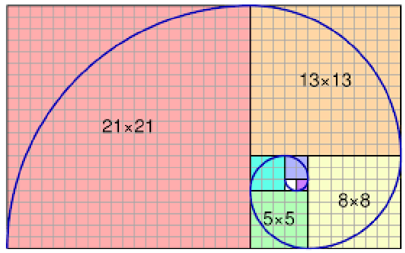
Application of Golden ratio: The **Golden Ratio** is a common mathematical **ratio** found in nature, which can be used to create pleasing, organic-looking compositions in your design projects or artwork

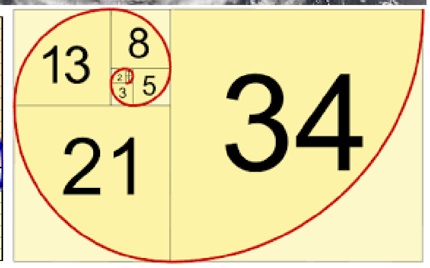
- Some examples: Flower petals. number of petals in a flower is often one of the following numbers: 3, 5, 8, 13, 21, 34 or 55. ...
- Faces. Faces, both human and nonhuman, abound with examples of the Golden Ratio. ...
- Body parts. ...
- Seed heads. ...
- 5. Fruits, Vegetables and Trees. ...
- Shells. ...
- Spiral Galaxies. ...
- Hurricanes.

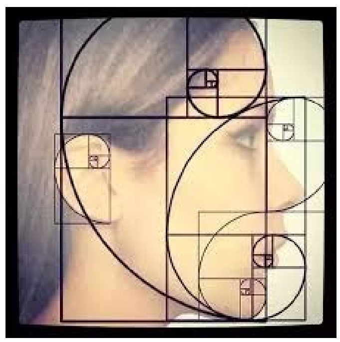
F(n)	F(n-1)	F(n)/F(n-1)
1	1	1
2	1	2
3	2	1.5
5	3	1.666666667
8	5	1.6
13	8	1.625
21	13	1.615384615
34	21	1.619047619
55	34	1.617647059
89	55	1.618181818

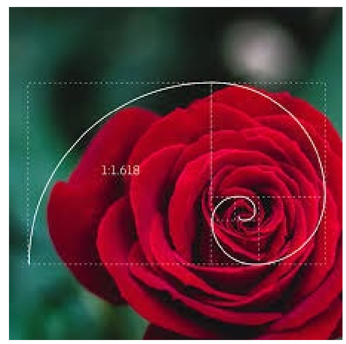


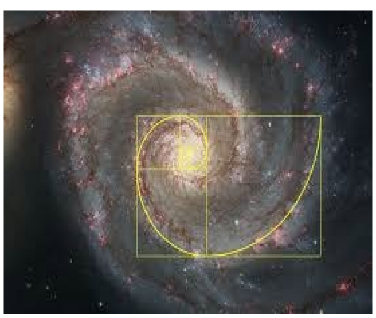


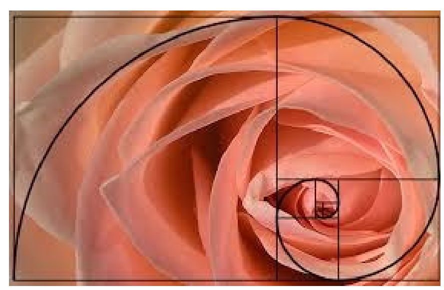


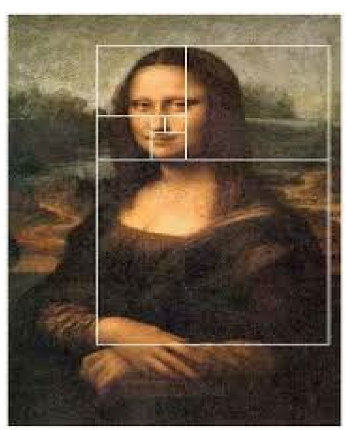












Section_______, Date________, Name____________________

Discovering Golden Ratio

					Round to the whole Number
F1			Round to 3 Decimal	F1	Use the x(golden raio)
F2		F2/F1		F2	
F3		F3/F2		F3	$\frac{x^3}{\sqrt{5}}=$
F4		F4/F3		F4	$\frac{x^4}{\sqrt{5}}=$
F5		F5/F4		F5	$\frac{x^5}{\sqrt{5}}=$
F6		F6/F5		F6	$\frac{x^6}{\sqrt{5}}=$
F7		F7/F6		F7	$\frac{x^7}{\sqrt{5}}=$
F8		F8/F7		F8	$\frac{x^8}{\sqrt{5}}=$
F9		F9/F8		F9	$\frac{x^9}{\sqrt{5}}=$
F10		F10/F9		F10	$\frac{x^{10}}{\sqrt{5}}=$
F11		F11/F10		F11	$\frac{x^{11}}{\sqrt{5}}=$
F12		F12/F11		F12	$\frac{x^{12}}{\sqrt{5}}=$
F13		F13/F12		F13	$\frac{x^{13}}{\sqrt{5}}=$
F14		F14/F13		F14	$\frac{x^{14}}{\sqrt{5}}=$
F15		F15/F14		F15	$\frac{x^{15}}{\sqrt{5}}=$

Please write what the nembers on the last column approaches to?

Golden Ratio = X=

See if you can draw an spiral by using numbers in Fibonacci Sequance

1, 1, 2, 3, 5,8,13,21