Principles of Counting

Objective: To find the total possible number of arrangements (ways) an event may occur.

a) Identify **the purpose** (Area Codes, Zip Codes, License Plates, Password, Short Melodies)
b) Number of parts for that purpose **(area code has 3 parts, zip code has 5 parts, pin number has 4 parts)**
c) What will go into each part (letter, digit, symbols, specific value or character)
d) How many choices are available **for each part?**
e) Finally multiply the number of choices!

1) How many different zip codes are possible? $\underline{D}\ \underline{D}\ \underline{D}\ \underline{D}\ \underline{D} = 10\times10\times10\times10\times10 = 100,000$

2) How many different zip codes are possible with no zero at the beginning?

$$\underline{D}\ \underline{D}\ \underline{D}\ \underline{D}\ \underline{D} = 9\times10\times10\times10\times10 = 90,000$$

3) How many different 7- part license plates are possible with one digit first, 3 letters after followed by another 3 digits?

$$\underline{D}\ \underline{L}\ \underline{L}\ \ \underline{L}\ \underline{D}\ \underline{D}\ \underline{D} = 10\times26\times26\times26\times10\times10\times10 = 175,760,000$$

4) How many different 7- part license plates are possible if each part can use letter or digit?

$$\underline{D}\ \underline{L}\ \underline{L}\ \ \underline{L}\ \underline{D}\ \underline{D}\ \underline{D} = 36\times36\times36\times36\times36\times36\times36 = 78,364,164,096$$

5) How many different 6-part password can be written (case sensitive with 10 digits, 52 letters and 8 symbols)

$$70\times70\times70\times70\times70\times70 = 117,649,000,000$$

6) How many different 12-note melodies can be made by a 44-key keyboard?

$$44^{12} = 52,654,090,776,777,588,736$$

7) How many different 4- digit even numbers can we write with (0,5,6,3,8,7)?

$$\underline{D}\ \ \underline{D}\ \ \underline{D}\ \ \underline{D}\quad 5\times6\times6\times3 = 540$$

Hint: To be 4- digit **zero can not be used** as the first digit, and to be an even number the **last numbe**r can be 0,6,8, that give us 3 choices.

8) A car model comes with the following choices:9 colors, with or without air conditioning, with or without sunroof, with or without automatic transmission, with or without a spoiler, and with or without antilock brakes. In how many ways can the car be ordered? The choices are:

Color AC/not sunroof/not transmission/not spoiler/not antilock/not

$$9\times2\times2\times2\times2\times2 = 288$$

9) You are about to take an8question multiple choice test. Each of these questions has4 answers (A, B, C, or D). How many ways can you answer the test if you leave an answer for each question?
8 questions or 8 parts and each one with 4 choices.

$$4\times4\times4\times4\times4\times4\times4\times4 = 4^8 = 65,536$$

10) A menu has 6 different sandwiches, with 3 choices of potato, 3 types of salad, and 5 different beverages. How many different lunches can be ordered consisting of a sandwich, potato, salad, and beverage?

$$6\times3\times3\times5 = 270$$

Factorial/Permutation/Combination

	$n!$	
1	$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$	
	$4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$	
	$3! = 3 \cdot 2 \cdot 1 = 6$	
	$2! = 2 \cdot 1 = 2$	
	$1! = 1 = 1$	
	$0! = ? = 1$	
	Why zero factorial is 1? $4! = \frac{5!}{5} = \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{5} = 4!$ $3! = \frac{4!}{4} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{4} = 3!$ $2! = \frac{3!}{3} = \frac{3 \cdot 2 \cdot 1}{3} = 2!$ $1! = \frac{2!}{2} = \frac{2 \cdot 1}{2} = 1!$ $0! = \frac{1!}{1} = \frac{1}{1} = 1$	
	Dividing factorials and use **shortcut** in simplifying those. You simplify by starting from **top number and going down by 1 before you hit the bottom numbe**r and **then you multiply them all.** $\frac{6!}{3!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 6 \cdot 5 \cdot 4 = 120$ $\frac{7!}{3!} = \frac{7.6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3 \cdot 2 \cdot 1} = 7 \cdot 6 \cdot 5 \cdot 4 = 840$ $\frac{100!}{97!} = 100 \cdot 99 \cdot 98 =$ $3!2! = ? = 3 \cdot 2 \cdot 1 \cdot 2 \cdot 1 = 12$ $3! + 2! = ? = 3 \cdot 2 \cdot 1 + 2 \cdot 1 = 8$	Remember that $\frac{6!}{3!} \neq 2!$
	Permutation $nP_x = \frac{n!}{(n-x)!}$ $5P_2 = \frac{5!}{(5-2)!} = \frac{5!}{3!} = 5 \cdot 4 = 20$ $8P_2 = \frac{8!}{(8-2)!} = \frac{8!}{6!} = 8 \cdot 7 = 56$ $18P_4 = \frac{18!}{14!} = 18 \cdot 17 \cdot 16 \cdot 15 =$	

	Combination $nC_x = \frac{n!}{x!(n-x)!}$ $5C_2 = \frac{5!}{2!(5-2)!} = \frac{5!}{2!\cdot 3!} = \frac{5\cdot 4}{2!} = \frac{20}{2} = 10$ $8C_3 = \frac{8!}{3!(8-3)!} = \frac{8!}{3!5!} = \frac{8\cdot 7\cdot 6}{3!} = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 8\cdot 7 = 56$ $8C_5 = \frac{8!}{5!(8-5)!} = \frac{8!}{5!3!} = \frac{8\cdot 7\cdot 6}{3!} = \frac{8\cdot 7\cdot 6}{3\cdot 2\cdot 1} = 56$ $11C_6 = 11C_5$	
	3 friends, 3 different gifts $3! =$	
	3 friends, 2 different gifts $3P_2 =$	
	3 friends, 2 of the same gifts $3C_2 =$	

Learn how to use you **calculator** to do **Factorial, Combination, and Permutation!!!!**

Factorial: Number of ways **n** objects or subjects can be arranged.

In how many ways 3 people can line up for a picture? $3! = 3 \cdot 2 \cdot 1 = 6$

ABC, ACB, BAC, BCA, CAB, CBA

In how many ways five people can line up for a picture? $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$

In how many ways can we arrange **3** books in a bookshelf? $3! = 3 \cdot 2 \cdot 1 = 6$

Permutation: Number of ways **x** objects **out of n** objects can be arranged when order in selection matters
The keywords are: arranged, ranked, lineup,…

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1 The ski club with **ten** members is to choose three officers **captain**, **co-captain** & **secretary**, how many ways can those offices be filled?

$${}_{10}P_3 = \frac{10!}{(10-3)1} = \frac{10!}{7!} = 10*9*8 = 720$$

2. Suppose you are asked to list, in order or preference, the three best movies you have seen this year. If you saw 10 movies during the year, in how many ways can the three best be chosen and **ranked**?

$${}_{10}P_3 = \frac{10!}{(10-3)1} = \frac{10!}{7!} = 10*9*8 = 720$$

3. In the Long Beach Air Race six planes are entered and there are no ties, in how many ways can the **first three** finishers come in?

$${}_{6}P_3 = \frac{6!}{(6-3)1} = \frac{6!}{3!} = 6*5*4 = 120$$

Combination: Number of ways **x** objects **out of n** objects can be arranged

TI-83/84 *n → math → PRB → Option 3 → x*

In how many ways can we select **two** out of **five** letters? ${}_5C_2 = \frac{5!}{2!3!} = 10$ **ways**

AB, AC, AD, AE, BC, BD, BE, CD, CE, DE

$${}_6C_1 = \frac{6!}{1!5!} = 6 \qquad {}_5C_4 = 5 \qquad {}_8C_4 = \frac{8!}{4!4!} = 70 \qquad {}_4C_2 = \frac{4!}{2!2!} = 6 \qquad {}_5C_0 = \frac{5!}{0!5!} = 1 \qquad {}_5C_5 = \frac{5!}{5!0!} = 1$$

- In how many ways a teacher can select 5 of his 23 students for a fieldtrip? ${}_{23}C_5$ **Ans:** 33,649
- In how many ways can we select 3- member committee from a group of 8 people? ${}_8C_3$ **Ans:** 56