Probability Distribution

<table>
<tr><td colspan="2">X= Random Variable
A variable that its value is unknown and can be discrete (countable) or continuous (measurable).</td></tr>
<tr><td>Discrete (countable)</td><td>Continuous (measurable)</td></tr>
<tr><td>Examples

- Number of applicants passing DMV test each day

- Number of traffic violations on campus each day

- Number of emergencies visits each day at Hospital</td><td>Examples

- Average rainfall each year in Sacramento

- Length of each newborn baby

- Height of each redwood tree</td></tr>
<tr><td>Different types of discrete probability distribution

- General discrete type
Expected Value = mean $= \mu = \sum\left(x\, p(x)\right)$
Standard deviation $= \sigma = \sqrt{\sum x^2 p(x) - \mu^2}$
- Binomial
Expected Value = mean $= \mu = np$
Standard deviation $= \sigma = \sqrt{np(1-p)}$</td><td>Different types of continuous probability distribution

- Uniform distribution

- Normal probability distribution</td></tr>
</table>

Example 1: There are 10 marbles in a bag, 5 reds, 3 greens and 2 blues. To play the game you pick one marble at random and if it is red, you get \$1, if it is green, you get \$2 and if it is blue, you pay \$5.
In this experiment the random variable is the dollar value corresponding to each color that you pick.
The objective is to construct a probability distribution table and compute the expected value.

Color of marble	X = random value	$p(x)$	$x\, p(x)$
Red	**1**	$5/10 = .5$	$1(0.5) = .50$
Green	**2**	$3/10 = .3$	$2(0.3) = .60$
Blue	**-5**	$2/10 = .2$	$-5(0.2) = -1.0$
			Find expected value $= \mu = \sum\left(x\, p(x)\right)$ by adding the numbers in this column $= \$0.10$

It means that we the more we play this game on the average we win \$0.10 per game.

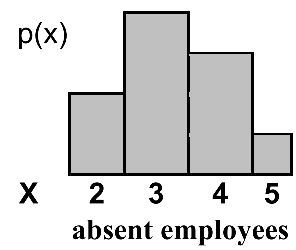
Example 2. Let **random variable** = **X** to be the number of **absent employees** in an office in each day.

X	f (days)	$p(x)$	$x\,p(x)$
2	10		
3	20		
4	15		
5	5 +		

Note that in this problem, we are given frequency but the probability for each outcome.
To find **probability values** $p(x)$ **in the 3rd column** divide each frequency by their sum in this case 50
To draw **probability distribution** use **x values** as **x- axis** and $p(x)$ values as **y-axis**.
To find the mean (**expected value**) create last column $x\,p(x)$ **by multiplying** x **and** $p(x)$ in each row. The mean (**expected value**) is the summation of $x\,p(x)$ column.

X	f (days)	P(X) $= f \div n$	$x\,p(x)$
2	10	$10/50 =$ 0.20	0.40
3	20	0.40	1.20
4	15	0.30	1.20
5	5 +	0.10 +	0.50 +
	$n =$ 50	**1.0 ?**	***3.3***
			Mean $= \mu = \sum(x\,p(x))$ ***= 3.3***

Probability Distribution.

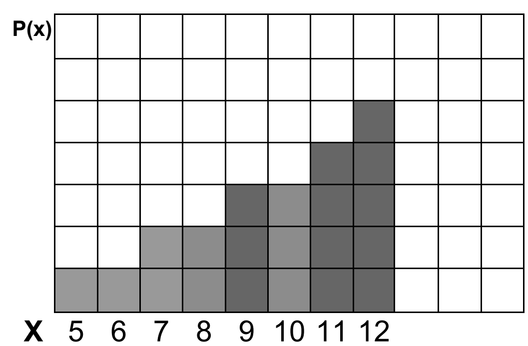


It is **most likely** that 3 employees will be absent/day
It is **least likely** that 5 employees will be absent/day.

1. Find the probability that there will be **at least** 4 **absent** in each day. 0.30 + 0.10 =.40
2. Find the probability that there will be **at most absent 4** in each day. 0.30 + 0.40 + 0.20 = .90
3. Find the **expected** number of number of absentees in each day. **Mean** $= \mu = \sum xp(x) = 3.3$

Example 3. Let **random variable = X** = the number of **reported car accidents** at Sun City in each day.

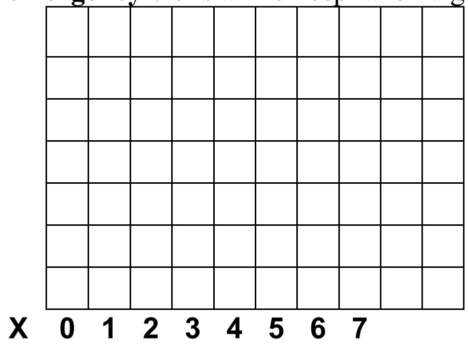
x	f	$p(x)$ %	$x\,p(x)$
5	2	.02=2%	0.10
6	3		
7	8		
8	9	.09=9	0.72
9	15		
10	18	.18=18	1.8
11	20		
12	25	.25 +	3 +
	100	1.0 ?	***Mean = ?***



- Complete the table and draw probability distribution and find the probability that.
1. At **least** there will be 10 returned accidents in each day. **Ans: 63 %**
2. At **most** there will be 7 returned accidents in each day. **Ans: 13 %**
3. Find the **expected number** of accidents in each day. ***Mean =9.91***

Example 4. Let **random variable** = **X** = the number of **emergency visits** at the hospital on a given day.

x	f	$p(x)$ %	$x\,p(x)$
0	2	2/64	0(2/64)
1	17		
2	10		
3	11		
4	10		
5	4		
6	8	8/64	6(8/64)
7	2		
		?	***Mean = ?***



- Complete the table, draw probability distribution, and find the probability that,

1. At **least** there will be 5 emergency visits in each day. **Ans: 22 %**
2. At **most** there will be 3 emergency visits in each day. **Ans: 63 %**
3. Find the **expected number** of emergency visits in each day. ***Mean = 3.00***

Expected Value Problems **Hint: To find the expected value use the formula** $\sum (x \times p(x))$

A. A \$1 slot machine in a casino has a winning prize of \$6 for each play with winning probability $15/100$. What are the expected results for the player and the house each time the game is played?

Outcome	x	$p(x)$	$x\,p(x)$
Win	6-1	$15/100$	$5 \times .15 = .75$
Lose	-1	$85/100$	$-1 \times .85 = -.85$
		$\sum p(x) = 1$	$\sum xp(x) = -0.10$

Hint: You subtract \$1 from \$6 of winning, because you are paying to play the game.
- Each time the game is played, player has an expected loss of \$.10 and the house an expected gain of \$.10
- If a slot machine is played 1000 times a day and 360 days a year then each machine is expected to generate revenue of $1000 \times 360 \times .10 = \$36,000$ per year. If a typical casino has 100 slot machines, then the total revenue will be $\$36,000 \times 100 = \$3,600,000$!!!!

B. A \$1 slot machine in a casino has a winning prize of \$6 for each play with winning probability $10/100$. What are the expected results for the player and the house each time the game is played?

Outcome	x	$p(x)$	$x\,p(x)$
Win	6-1	$10/100$	$5 \times .10 = .5$
Lose	-1		
		$\sum p(x) = 1$	$\sum xp(x) = -0.40$

How much will be the expected to generate revenue if a typical casino has 100 slot machines and each slot machine is played 1000 times a day and 360 days a year. **Ans: \$14,400,000 per year**

C) In a game, you have a 4 probability of winning \$100 and a 46 probability of losing \$10. What is your expected value?

Outcome	x	$p(x)$	$x\,p(x)$
Win	100	4/50	$100 \times .8 = 8$
Lose	-10	$46/50$	?
		$\sum p(x) = 1$	$\sum xp(x) = -1.2$

Hint: You **do not** subtract \$10 from \$100 of winning, because you are not paying to play the game.

D) A contractor is considering a sale that promises a profit of $20,000 with a probability of 0.60 or a loss (due to bad weather, strikes, and such) of $10,000 with a probability of 0.4. What is the **expected profit**?

Outcome	x	$p(x)$	$x\,p(x)$
profit	$20,000	0.6	$12,000
loss	-$10,000	0.4	-$4,000
		$\sum p(x)=1$	$\sum xp(x)=$ $8,000

E) Suppose you buy 1 ticket for $1 out of a lottery of 1000 tickets where the prize for the one winning ticket is to be $5000. What is your expected value?

Outcome	x	$p(x)$	$x\,p(x)$
Win	5000-1	1/1000	
Lose	-1	999/1000	
		$\sum p(x)=1$	$\sum xp(x)=$

A) $40.00 **B)** $4.00 **C)** $0.40 **D)** $-$0.40

F) A 28-year-old man pays $159 for a one-year life insurance policy with coverage of $140,000. If the probability that he will live through the year is 0.9994, what is the expected value for the insurance policy? **5)** _____

A) -$158.90 **B)** $139,916.00 **C)** $-$75.00 **D)** $84.00

G) On a multiple-choice test, a student is given five possible answers for each question. The student receives 1 point for a correct answer and loses ¼ point for an incorrect answer. If the student has no idea of the correct answer for a particular question and merely guesses, what is the student's expected gain or loss on the question? **7)** ______

Outcome	x	$p(x)$	$x\,p(x)$
Correct	1	1/5	
Incorrect	-0.25	4/5	
		$\sum p(x)=1$	$\sum xp(x)=$

A) 0 **B)** 0.25 **C)** 0.133 **D)** -0.33

H) Suppose that on any of the questions you can eliminate two of the five answers as being wrong. If you guess at one of the remaining three answers, what is your expected gain or loss on the question? **8)** _____

Outcome	x	$p(x)$	$x\,p(x)$
Correct	1		
Incorrect	-0.25		
		$\sum p(x)=1$	$\sum xp(x)=$

A) 0 **B)** 0.167 **C)** 0.133 **D)** 0.63

E-Lottery		
x	p(x)	x . P(x)
4999	0.001	4.999
-1	0.999	-0.999

F- Life Insurance			
	x	p(x)	x . P(x)
Die	140000	0.0006	84
Survive	-159	0.9994	-158.9046

1	4

1	-74.9046

G- Multiple choice			
	X	P(x)	X*P(X)
Correctly	1	0.2	0.2
Incorrectly	-0.25	0.8	-0.2
		1	0

H- Multiple choice			
	X	P(x)	X*P(X)
Correctly	1.000	0.333	0.333
Incorrectly	-0.250	0.667	-0.167
		1	0.167