Part Two (Section 6) **Binomial Probability**

4 Assumptions are required

1. Each trial must have only **two outcomes.**
   Examples: **P**ass/**F**ail on DMV test**,** A newborn baby to be **B**oy or **G**irl**,** guess **correctly** or **not,** getting **a** **T**ail or **H**ead
2. The **probability** must remain **constant** for each trial.
   ***Example*:** when you flip a coin there is always 50% chance to be tail or when guessing a multiple-choice question with 4 choices then there is always a constant 1/4 chance to be correct.

3.The **trials** must be **independent**. The outcome of each trial is independent from other trials
Example: Each guess may turn to be correct or incorrect with respect of other guesses. The result of each flip is independent of other flips.

4. The experiment should have a **fixed number of trials**.
   Example: Number of questions to be guessed or number of times you flip a coin.

Binomial Probability formula $P(x) = {}_nCx\ p^x(1-p)^{n-x}$ $p = probability\ of\ Success$

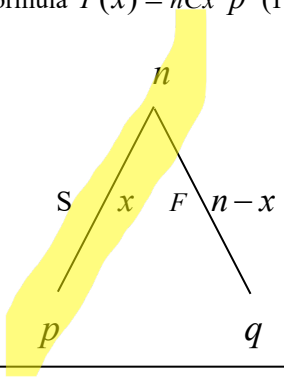
$n = Total\ number\ of\ trials$ $x = Number\ of\ success\ outcomes$

$nCx = \dfrac{n!}{x!(n-x)!}$ $number\ of\ ways\ success\ may\ occure.$

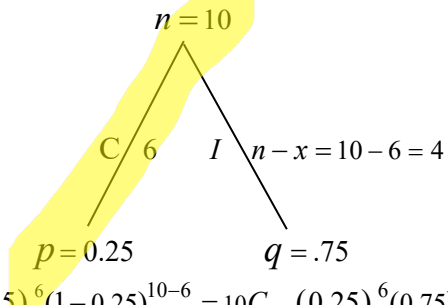
Mean or Expected values $= \mu = n\,p$ St. Dev. $= \sigma = \sqrt{n\,p(1-p)}$

Hint: To find binomial probability, for each case

1. Draw a triangle, put **n** = number of subjects or observation at the **top**,
2. Label each branch as success (what the question is) and failures (the opposite).
3. Put x = number of successes on the left branch and $n - x$ on the right branch
4. Put **p** (probability of success) at the **bottom of left branch** and **q = 1- p** (probability of failure) at the **bottom of right branch.**
5. Use the formula $P(x) = {}_nCx\ p^x(1-p)^{n-x}$.



Example 1: If you guess all 10 multiple choice quiz (4 choices for each question), What is the probability that you **exactly guess 6 questions correctly**? Success is to guess correctly so $p = 1/4 = 0.25$ and failure is to guess incorrectly $q = 3/4 = 0.75$



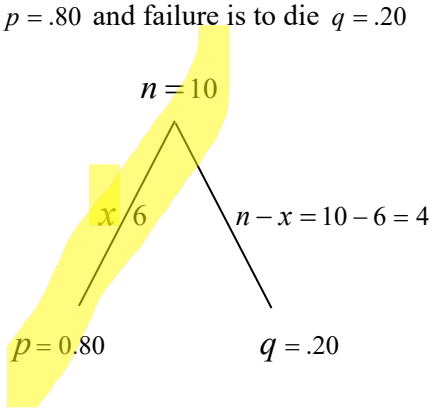
$${}_{10}C_6\ (0.25)^6(1-0.25)^{10-6} = {}_{10}C_6\ (0.25)^6(0.75)^4 = 210(.000244)(.3164) = 0.0162 = 1(1)(.4219) = \mathbf{0.01621}$$

By TI 83/84 = binompdf*(10,6,.25) = 0.01621*

What is the **expected or average/mean** number of corrected guesses? $= \mu = np = 10(1/4) = 10(.25) = 2.5$

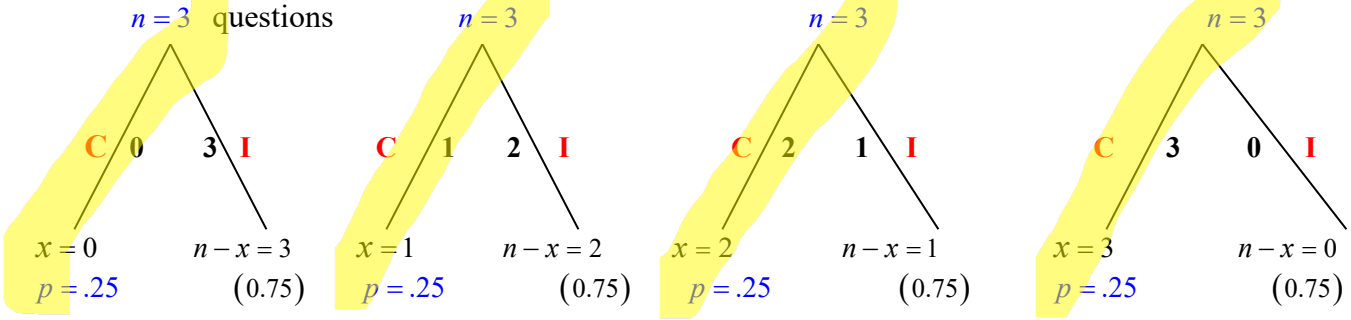
What is the **standard deviation** of corrected guesses? $= \sigma = \sqrt{npq} = \sqrt{10(1/4)(3/4)} = \sqrt{10(.25)(.75)} = 1.369$

Example 2. Hospital records show that of patients suffering from a certain disease, 20% die of it. What is the probability that of 6 randomly selected patients, 6 will recover? **Random Variable =X= ? = number of patients recovered.** Success is to recover so $p = .80$ and failure is to die $q = .20$



$_{10}C_6\,(0.80)^6(1-0.80)^{10-6} = 210(0.2621)(0.0016) = 0.0881$ or use TI 83 **binompdf*(10,080.6) = 0.0881***

Example 3. John wants to guess 3 multiple choice questions (each question has 4 choices).
So the **random variable = X = number of correct guesses that he can make and they can be can be 0,1,2,3**
$n = 3$ is number of questions to be guessed and $p = 1/4 = 0.25$ is the chance to guess each question correctly and $1 - p = 3/4 = 0.75$ is the chances to guess incorrectly. So To find $P(0)$, probability that **no question** to be guessed correctly can be found by setting the triangle, label all necessary information and then use the formula
$P(x) = nCx\ p^x(1-p)^{n-x}$ $3C0\,(0.25)^0(1-0.25)^{3-0} = 3C0\,(0.25)^0(0.75)^3 = 1(1)(.4219) =$ **0.4219** or **TI 83/84**
For each triangle $n = 3$, $p = 1/4 = .25$ and **X** will be changed to $= 0, 1, 2, 3$ Also **C:Correct I:Incorrect**



The probability that **no one correct** $= 3C0\,(0.25)^0(1-0.25)^{3-0} = 3C0\,(0.25)^0(0.75)^3 = 1(1)(.4219) =$ **0.4219**

X	P(X)	$p(x)$ **by TI 83/84**	$x\,p(x)$
0	$3C0\,(0.25)^0(1-0.25)^{3-0} = 3C0\,(0.25)^0(0.75)^3 = 1(1)(.4219) =$ **0.4219**	**binompdf*(3,0.25,0) = 0.4219***	**0**
1	$3C1\,(0.25)^3(1-0.25)^{3-1} = 3C1\,(0.25)^1(0.75)^2 = 3(.25)(.5625) =$ **0.4219**	**binompdf*(3,0.25,1) = 0.4219***	**.4219**
2	$3C2\,(0.25)^2(1-0.25)^{3-2} = 3C2\,(0.25)^2(0.75)^1 = 3(.625)(.75) =$ **0.1406**	**binompdf*(3,0.25,2) = 0.1406***	**.2812**
3	$3C3(0.25)^3(1-0.25)^{3-3} = 3C3\,(0.25)^3(0.75)^0 = 1(.512)(1) =$ **0.0156**	**binompdf*(3,0.25,3) = 0.0156***	**0.4688**

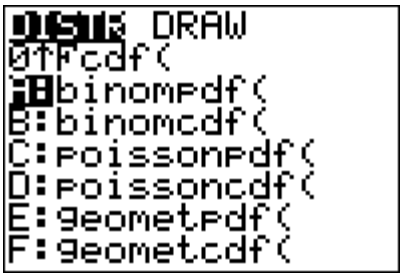
Mean $= \sum x\,p(x) = .75$ **or *short- cut*** $\mu = np = 3(.25) = 0.75$

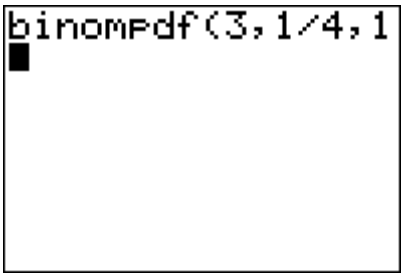
Based on the above table, find the probability that
1. All three will be correct. **P(X = 3) = 0.0156**
2. None will be correct. **P(X = 0) = 0.4219**
3. At least 2 will be correct. $0.1406 + 0.0156 = 0.1562$
4. At most 1 will be correct. $.4219 + .4219 = 0.8438$
5. Expected number of correct answers. $\mu = np = 3(.25) = 0.75$
6. Standard deviation of correct answers. $\sigma = \sqrt{np(1-p)} = \sqrt{3(.25)(1-.25)} = .75$

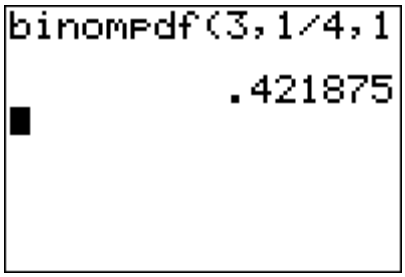
TI-83/84

Using TI 83/84 to solve the problem from last page.
For example to find the probability for **x = 1,** we can use **TI 83/84** as such

2nd Distribution, select binompdf type 3,1/4,1 press enter



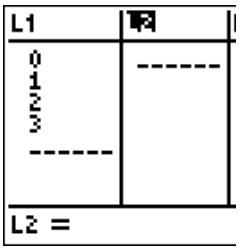


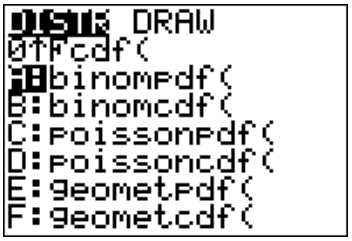


And you can continue, the same way for getting the probability for x = 1, 2, 3

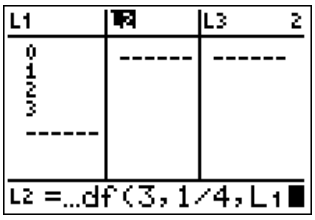
If you want TI to calculate all the probabilities for x=0,1,2,3 then the following short cut will be very helpful

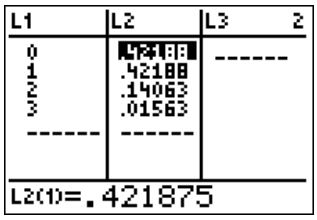
Enter 0,1,2,3 in L1 go to the very top of L2 2nd Distribution, select binompdf



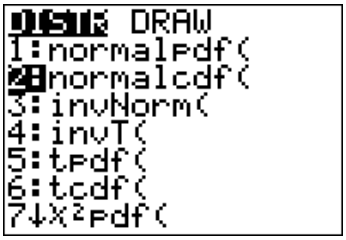


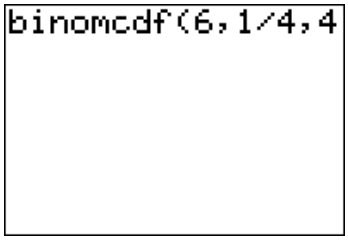
Enter ***3,*** $1 \div 4$ ***,L1*** *and then enter* ***Answers will now be listed on L2***

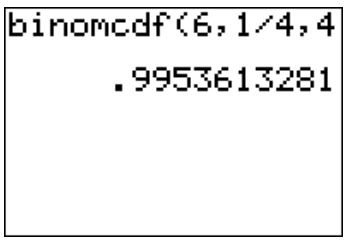




Example 3. Find the probability that out of 6 multiple questions at most 4 are guessed correctly. The short cut is







Example 4. According to Abe, **80**% of his students pass his stat class, if **5** of his students are randomly selected and **random variable** = **X = number of his students that will pass his stat class,** then complete the probability distribution table,

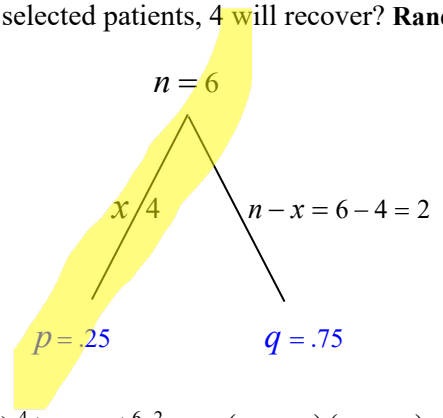
X	P(X)	$p(x)$ **by TI 83/84**
0	$_5C_0\,(0.80)^0(1-0.80)^{5-0} = 1(1)(.00032) =$ **.00032**	**binompdf***(5,0.80,0)* **= *0.00032***
1	$_5C_1\,(0.80)^1(1-0.80)^{5-1} = 5(.8)(.0016) =$ **.0064**	**binompdf***(5,0.80,1)* **= *0.0064***
2	$_5C_2\,(0.80)^2(1-0.80)^{5-2} = 10(.64)(.008) =$ **.0512**	**binompdf***(5,0.80,2)* **= *0.0512***
3	$_5C_3\,(0.80)^3(1-0.80)^{5-3} = 10(.512)(.04) =$ **.2048**	**binompdf***(5,0.80,3)* **= *0.2048***
4	$_5C_4\,(0.80)^4(1-0.80)^{5-4} = 5(.4096)(.2) =$ **.4096**	**binompdf***(5,0.80,4)* **= *0.4096***
5	$_5C_5\,(0.8)^5(1-0.80)^{5-5} = 1(0.32768)(1) =$ **.32768**	**binompdf***(5,0.80,5)* **= *0.32768***

Based on above table, find the probability that

1. All lucky five will pass. **= .32768**
2. None will pass. = **.00032**
3. At least 3 will pass. **.2048 +.4096+.32768 = .94208**
4. At most 3 will pass. **.00032+.0064+.0512+.2048 =.26272**
5. Expected number of students that will pass. $\mu = np = 5(.80) = 4$
6. Standard deviation of number of students that will pass. $\sigma = \sqrt{np(1-p)} = \sqrt{5(.80)(1-.80)} = 0.8944$

More Practices for Binomial Probability

1. Hospital records show that of patients suffering from a certain disease, 75% die of it. What is the probability that of 6 randomly selected patients, 4 will recover? **Random Variable =X= ? = number of patients recovered**



$_6C_4\,(0.25)^4(1-0.25)^{6-2} = 15(0.0039)(0.5625) = 0.0329$ or use TI 83 **binompdf***(6,0.25.4)* **= *0.03296***

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2. Hospital records show that of patients suffering from a certain disease, 75% die of it. What is mean of expected number of patients will recover? **Random Variable =X= ? = number of patients recovered**
binompdf*(6,0.25.4)* **= *0.03296***

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3. In the old days, there was a probability of 0.8 of success in any attempt to make a telephone call.

Calculate the probability of having 7 successes in 10 attempts. **Random Variable =X= ? number of success**

binompdf*(10,0.8.7)* **= *0.20133***

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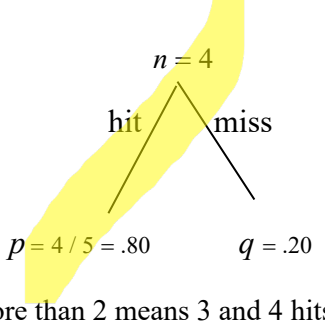
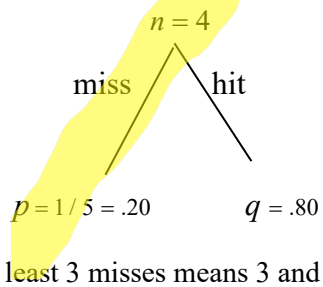
4. A (blindfolded) marksman finds that on the average he hits the target 4 times out of 5. If he fires 4 shots, what is **Random Variable =X= ? = number of hits** and what is the probability of

(a) more than 2 hits? **Success is hit** so $p = 4/5 = 0.8$ and **failure is miss** $q = 1/5 = 0.2$



More than 2 means 3 and 4 hits: **binompdf***(4,4/5.3)* + **binompdf***(4,4/5.4)* ***= 0.8192***

b) at least 3 misses? **Success is miss** so $p = 1/5 = 0.2$ and **failure is hit** $q = 4/5 = 0.8$



At least 3 misses means 3 and 4 misses: **binompdf***(4,1/5.3)* + **binompdf***(4,1/5.4)* ***= 0.0272***

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5) A quiz consists of 10 true false questions. To **pass** the quiz a student must **get 70%** or better on the quiz. If a student randomly guesses, what is the probability that the student will pass the quiz?

To pass student must answer 7 or 8 or 9 or 10 questions correctly.

binompdf*(10,1/2,7)* + **binompdf***(10,1/2,8)* + **binompdf***(10,1/2,9)* + **binompdf***(10,1/2,10)* = ***0.1719***

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6) A quiz consists of 10 multiple choice questions, each with five possible answers, one of which is correct. To **pass** the quiz a student must get **70%** or better on the quiz. If a student randomly guesses, what is the probability that the student will pass the quiz? **To pass student must answer 7 or 8 or 9 or questions correctly.**

binompdf*(10,1/5,7)* + **binompdf***(10,1/5,8)* + **binompdf***(10,1/5,9)* + **binompdf***(10,1/5,10)* = ***0.0009***

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7. A multiple choice test contains 20 questions. Each question has five choices for the correct answer. Only one of the choices is correct. What is the probability of making an 80 with random guessing? ***Ans: 0.000000013***

Random Variable =X= ? number of correct guesses

To het 80% means to get 16 out of 20 correct **binompdf***(20,1/5.16)* **= 0.000000013**

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8) A study indicates that 4% of American teenagers have tattoos. You randomly sample 30 teenagers. What is the likelihood that exactly 3 will have a tattoo? ***Ans: 0.0863***

Random Variable =X= ? = number teenagers with tattoos **binompdf***(30,0.04,3)* ***= 0.0863***

9. A manufacturer of metal pistons finds that on the average, 12% of his pistons are rejected because they are either oversize or undersize. What is the probability that a batch of 10 pistons will contain

Random Variable =X= ? = number of rejected pistons

a) no more than 2 rejects? **binompdf***(10,0.12,0)* + **binompdf***(10,0.12,1)* + **binompdf***(10,0.12,2)*= **0.8913**

b) at least 2 rejects? **1-(binompdf***(10,0.12,0)* + **binompdf***(10,0.12,1)* = **(1-0.65827) =0.34173**

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10) Suppose a die is tossed 5 times. What is the probability of getting exactly 2 fours?

Random Variable =X= number of times getting 4 when toss a die 5 times ***Ans:*** **binompdf***(5,1/6,2)* = ***0.161***

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11) Find the **mean** for the number of sixes that appear when rolling 30 dice.

Random Variable =X= number of times getting 6 when toss 30 dice ***Ans:*** $\mu = np = 30(1/6) =$ ***5***

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12) Knowing that about 12% of people are left-handed,

a) Find the probability of that out of 25 students 5 are left-handed. ***Ans:*** **binompdf***(25,0.12,5)* = ***0.103***

b) How many are expected to be left-handed? ***Ans:*** $\mu = np = 25(0.12) = 3$

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13) Find the mean for the number of corrected answers on a 20 multiple choice questions (5 choices), if all answers were guessed. ***Ans:*** **Random Variable =X= ?** $\mu = np = 20(1/5) = 4$

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14) A company owns 400 laptops. Each laptop has an 8% probability of not working. You randomly select 20 laptops for your salespeople. **Random Variable =X= ?**

(a) What is the likelihood that 5 will be broken? **binompdf***(20,0.08,5)* = ***0.0145***
(b) What is the likelihood that they will all work? **binompdf***(20,0.08,0)* = ***0.1887***
(c) What is the likelihood that they will all be broken? **binompdf***(20,0.08,20)* = **0.00000000000000000001**

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15) An XYZ cell phone is made from 55 components. Each component has a .002 probability of being defective. What is the probability that an XYZ cell phone will not work perfectly?

Random Variable =X= number of defective components? 1-binompdf*(55,0.002,0)* = **1-0.8957** = ***0.1043***

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16) The ABC Company manufactures toy robots. About 1 toy robot per 100 does not work. You purchase 35 ABC toy robots. What is the probability that exactly 4 do not work?

Random Variable =X= number of robots do not work? binompdf*(35,0.012,4)* = ***0.00038***