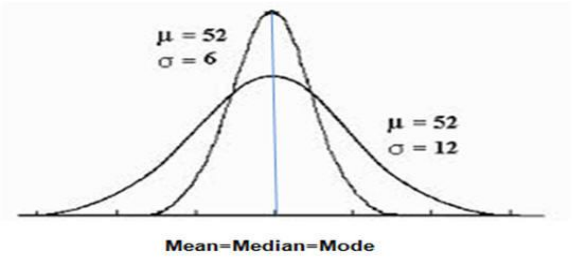
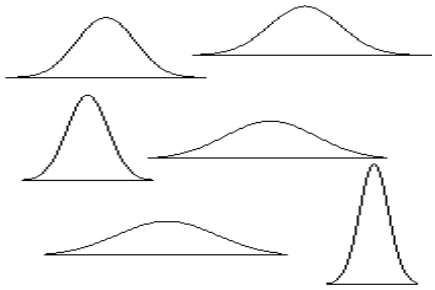
$$y = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(\bar{x}-\mu)^2/2\sigma^2}$$



Smaller standard deviation will result in narrower normal width.



Normal distributions are a family of distributions that have the same general shape. They are symmetric with scores more concentrated in the middle than in the tails. Normal distributions are sometimes described as bell shaped. Examples of normal distributions are shown above on the left. Notice that they differ in how spread out they are. The area under each curve is the same. The height of a normal distribution can be specified mathematically in terms of two parameters: the mean (μ) and the standard deviation (σ).

Properties

1. Normal Probability Distribution deals with continuous random variables.
   (age, speed, temp, weight, length, time, …)

2. The ***entire area*** under the curve is *100% = 1*, 50% of area to the left and 50 % to the right.

3. The **larger** the **standard deviation** the ***wider the distribution*** will be.

4. The ***area*** under the curve represents the ***probability***.

5. The ***graph of the standard normal curve approaches zero*** as z increases in positive direction or decreases in negative direction.

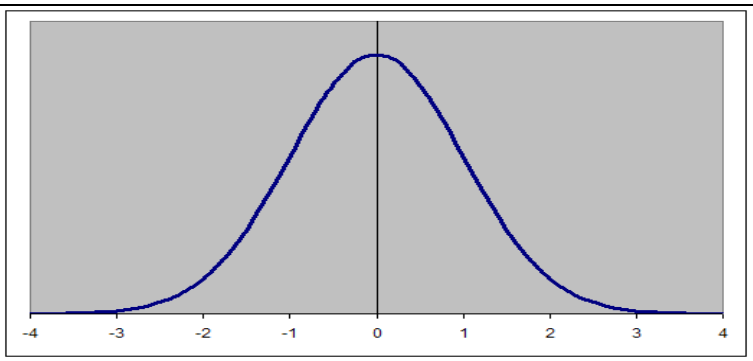
6. The ***area or percentage under the curve*** (area between two boundaries) can be about ***an individual*** or the ***entire population***.

Standard Normal Probability Distribution (SNPD)

It is a special case of normal distribution when $\mu = 0$ and $\sigma = 1$ the **horizontal axis is called the Z-axis**.

The graph of the standard normal curve approaches zero as z increases in positive direction or decreases in negative direction

$\mu = 0$ and $\sigma = 1$



Finding area (percentage) under Standard Normal Probability distribution by using TI 83/84

Note 1: When using TI 83/84,

You need a Lower Boundary *LB* *or, an* Upper Boundary *UB* *and* $\mu = 0$ and $\sigma = 1$

Note 3: Sketch a normal curve, draw both boundaries and shade the area in between the boundaries.

Note 4: If one boundary is missing either Lower or Upper, then use the following rule to create one.

Formulas to create **missing** Lower Boundary $LB = \mu - 5\sigma$
Formulas to create **missing** Upper Boundary $UB = \mu + 5\sigma$

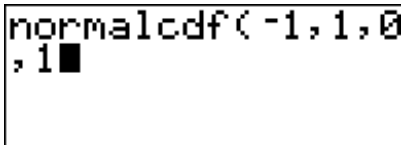
Steps to use TI-83/84

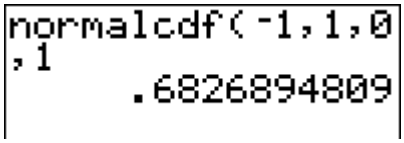
2nd → DISTR → Option 2 then normalcdf (LB,UB,0,1) → enter

Example 1 Find the area (percentage) between $z = -1$ and $z = 1$ $P(-1 < Z < 1) = ?$ (68% empirical rule)

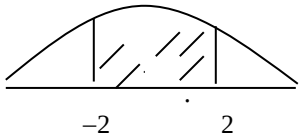
TI-83/84 ***2nd → DISTR → Option 2 then normalcdf (LB,UB,0,1) → enter***

TI-83/84 ***2nd → DISTR → Option 2 then normalcdf (–1,1,0,1) → enter*** **answer:** 68.27%





Example 2 Find the area (percentage) between $z = -2$ and $z = 2$ $P(-1 < Z < 2) = ?$ (**95%** empirical rule)

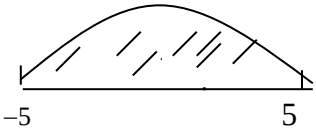


TI-83/84 ***2nd → DISTR → Option 2 then normalcdf (–2,2,0,1) → enter*** **answer:** 95.45%

Example 3 Find the area (percentage) between $z = -3$ and $z = 3$ (basically applying **99.7%** empirical rule)

TI-83/84 ***2nd → DISTR → Option 2 then normalcdf (–3,3,0,1) → enter*** **answer:** 99.73%

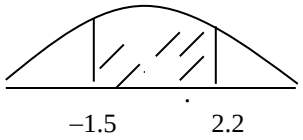
Example 4 Find the area (percentage) between $z = -5$ and $z = 5$ (between **5** standard deviation)
Important



TI-83/84 ***2nd → DISTR → Option 2 then normalcdf (–5,5,0,1) → enter*** **answer:** 99.99%

So, by **5 standard deviation on both sides** of the mean **almost 100%** of data are covered!

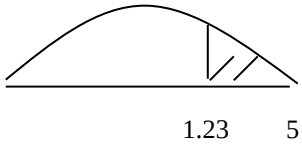
Example 5 Find the area (percentage) between $z = -1.5$ and $z = 2.2$ $P(-1.5 < Z < 2.2) = ?$



TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$-1.5, 2.2, 0, 1$***)*** $\rightarrow$ ***enter*** **answer:** 91.92%

Example 6 **Find the area (percentage) greater than** $z = 1.23$

$P(1.23 < Z) = ?$

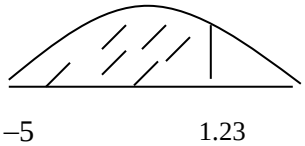


Upper boundary is missing: create an upper boundary $UB = \mu + 5\sigma$ **in this case** $UB = 0 + 5(1) = 5$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$1.23, 5, 0, 1$***)*** $\rightarrow$ ***enter*** **answer:** 10.93%

Example 7 **Find the area (percentage) less than** $z = 1.23$

$P(Z < 1.23) = ?$

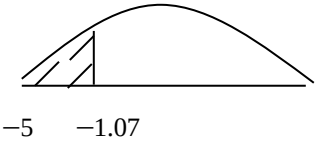


Lower boundary is missing: create a lower boundary $LB = \mu - 5\sigma$ **in this case** $LB = 0 - 5(1) = -5$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$-5, 1.23, 0, 1$***)*** $\rightarrow$ ***enter*** **answer:** 89.07%

Example 8 **Find the area (percentage) less than** $z = -1.07$

$P(Z < -1.07) = ?$

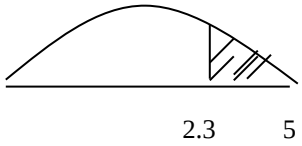


Lower boundary is missing: create a lower boundary $LB = \mu - 5\sigma$ **in this case** $LB = 0 - 5(1) = -5$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$-5, -1.07, 0, 1$***)*** $\rightarrow$ ***enter*** **answer:** 14.23%

Example 9 **Find the area (percentage) greater than** $z = 2.35$

$P(2.35 < Z) = ?$



Upper boundary is missing: create an upper boundary $LB = \mu + 5\sigma$ **in this case** $UB = 0 + 5(1) = 5$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$2.35, 5, 0, 1$***)*** $\rightarrow$ ***enter*** **answer:** 0.94%

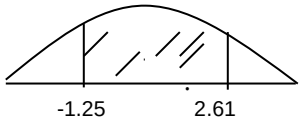
More Practice on S N P D when $\mu = 0$ and $\sigma = 1$ the **horizontal axis is Z-axis**.

TI-83/84 ***2nd → DISTR → Option 2 then normalcdf (LB,UB,0,1) → enter***

Formulas to create **missing** Upper Boundary $UB = \mu + 5\sigma$ $UB = 0 + 5(1) = 5$

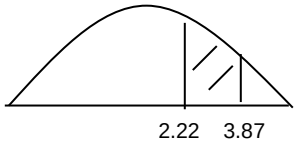
missing Lower Boundary $LB = \mu - 5\sigma$ $LB = 0 - 5(1) = -5$

1) P(-1.25 < Z < 2.61) =



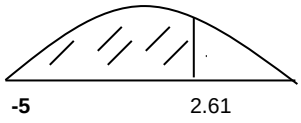
Answer = ***normalcdf (*** $-1.25, 2.61, 0, 1$ ***) = 0 .8899***

2) P(2.22 < Z < 3.87) =



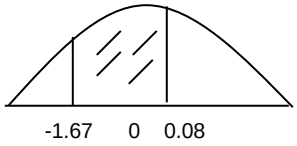
Answer = ***normalcdf (*** $2.22, 3.87, 0, 1$ ***) = 0 .0131***

3) P(Z < 2.61) =



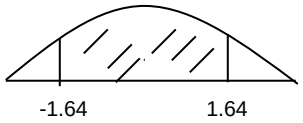
Answer = ***normalcdf (*** $-5, 2.61, 0, 1$ ***) = 0 .9955***

4) P(-1.67 < Z < 0.08) =



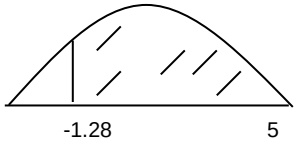
Answer = ***normalcdf (*** $-1.67, 0.08, 0, 1$ ***) = 0 .4844***

5) P(-1.64 < Z < 1.64) =



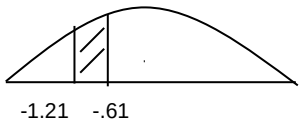
Answer = ***normalcdf (*** $-1.64, 1.64, 0, 1$ ***) = 0 .8990***

6) P(-1.28 < Z) =



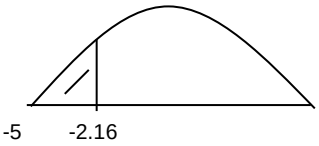
Answer = ***normalcdf (*** $-1.28, 5, 0, 1$ ***) = 0 .8997***

7) P(-1.21 < Z < -0.61) =



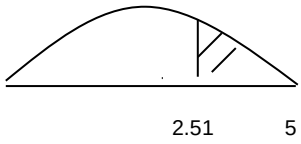
Answer = ***normalcdf (*** $-1.21, -0.61, 0, 1$ ***) = 0 .1578***

8) P(Z < -2.16) =



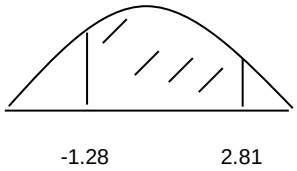
Answer = ***normalcdf (*** $-5, -2.16, 0, 1$ ***) = 0 .0154***

9) P(2.51 < Z) =



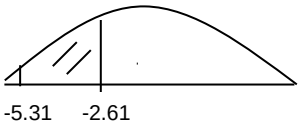
Answer = ***normalcdf (*** $2.51, 5, 0, 1$ ***) = 0 .0060***

10) P(-1.82 < Z < 2.81) =



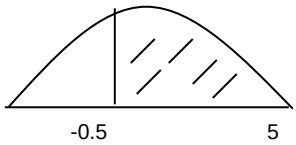
Answer = ***normalcdf (*** $-1.28, 2.81, 0, 1$ ***) = 0 .9631***

11) P(-5.34 < Z < -2.61) =



Answer = ***normalcdf (*** $-5.31, -2.61, 0, 1$ ***) = 0 .0044***

12) P(-0.5 < Z) =



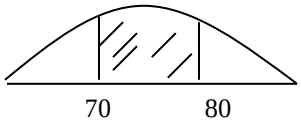
Answer = ***normalcdf (*** $-0.5, 5, 0, 1$ ***) = 0 .6915***

Non-Standard Normal Probability Distribution $\mu \neq 0$ **and** $\sigma \neq 1$

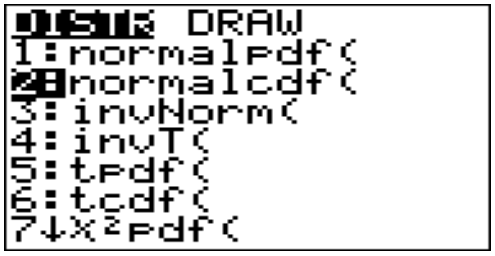
TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (LB,UB,*** μ, σ ***)*** $\rightarrow$ ***enter***

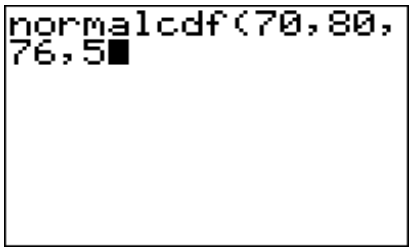
The average score for final stat exam was 76 with a standard deviation 5. If scores are normally distributed answer the following questions: **A normal distribution that** $\mu = 76,$ $\sigma = 5$ **and the horizontal axis is called the X-axis.**

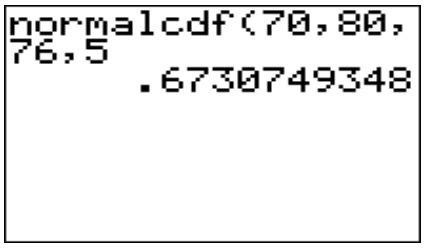
1. What percentage of students got scores between **70** and **80**?



TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$70, 80, 76, 5$***)*** $\rightarrow$ ***enter*** **answer: 67.31%**







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2. What percentage of students got scores between **80** and **90**?

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$80, 90, 76, 5$***)*** $\rightarrow$ ***enter*** **answer:** 20.93%

---

3. What percentage of students got scores less than **70**? Lower boundary is missing
In this case, the logical choice for Lower boundary is $LB = 0$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$0, 70, 76, 5$***)*** $\rightarrow$ ***enter*** **answer:** 11.51%

---

4. What percentage of students got scores more than **90**? Upper boundary is missing here., in this case**,** the logical choice for upper boundary is $UB = 100$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$90, 100, 76, 5$***)*** $\rightarrow$ ***enter*** **answer:** 0.26%

---

5. What percentage of students got scores **within** one standard deviation of the mean?
For this problem

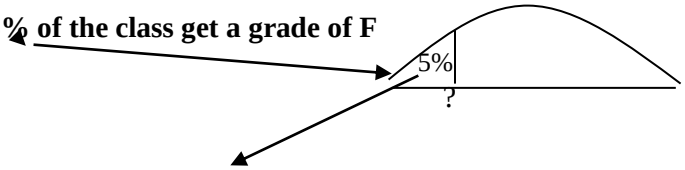
Lower boundary: $LB = \mu - 1\sigma = 76 - 5 = 71$ Upper boundary: $UB = \mu + 1\sigma = 76 + 5 = 81$

TI-83/84 ***2nd*** $\rightarrow$ ***DISTR*** $\rightarrow$ ***Option 2*** ***then*** ***normalcdf (***$71, 81, 76, 5$***)*** $\rightarrow$ ***enter*** **answer:** 68.27%

Finding the *cut-of point* with a given %

Finding cut-off point means, **given an area** either to the right or left, then **find its corresponding boundary**.
The final stat exam had an **average of 76** with a **standard deviation 5**. If scores are normally distributed answer the following questions

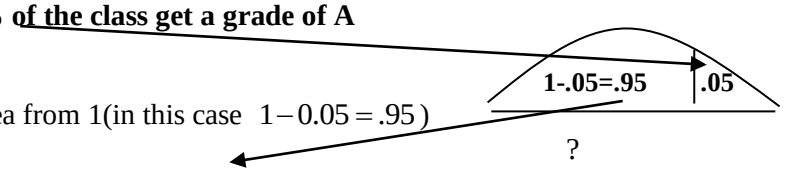
Ex:1 According to grading policy, the **bottom 5% of the class get a grade of F**
Find the cutting score for F



TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 3*** ***then*** ***invNorm*** $(0.05, 76, 5)$ $\longrightarrow$ ***enter*** **answer:** $x = 67.778$

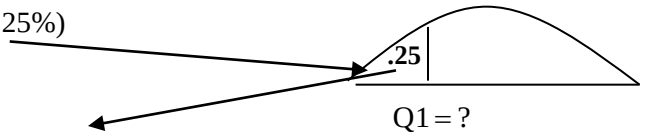
Ex: 2 According to grading policy, the **top 5% of the class get a grade of A**

In using TI, area on the top must be subtract area from 1(in this case $1-0.05=.95$)



TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 3*** ***then*** ***invNorm*** $(0.95, 76, 5)$ $\longrightarrow$ ***enter*** **answer:** $x = 84.22$

Ex: 3 Find the score that corresponds to the Q1(bottom 25%)



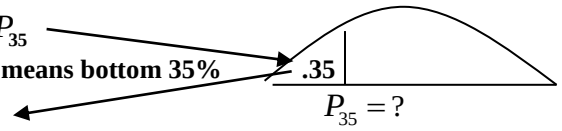
TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 3*** ***then*** ***invNorm*** $(0.25, 76, 5)$ $\longrightarrow$ ***enter*** **answer:** 72.63

Ex: 4 Find the score that corresponds to the Q3 (bottom 75% or top 25%)

In using TI, area on the top must be subtract area from 1(in this case $1-0.05=.95$)

TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 3*** ***then*** ***invNorm*** $(0.75, 76, 5)$ $\longrightarrow$ ***enter*** **answer:** $x = 79.37$

Ex:5 Find the score that corresponds to the 35TH Percentile $= P_{35}$
Hint: Percentile always refers to the bottom area, and in this case P_{35} means bottom 35%



TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 3*** ***then*** ***invNorm*** $(0.35, 76, 5)$ $\longrightarrow$ ***enter*** **answer:** $x = 74.07$

Application of Normal Probability Distribution

1) On a given test the average test scores was 68 with standard deviation of 8. If the scores are normally distributed, then find the probability as what percentage of students got scores,

a) Between 60 and 70? ***Answer*: *44.05%***

b) Between 70 and 80? ***Answer*: *33.45%***

c) Between 80 and 90? ***Answer*: *6.38%***

d) Less than 60? ***Answer*: *15.86%***

e) More than 90? ***Answer*: *0.29%***

f) Find the cut-off point for F if the bottom 1% will be getting "F". ***Answer*: *49.39***

g) Find the cut-off point for "A" if the top 2% will be getting "A" ***Answer*: *84.43***

h) Find the score for Q1 ***Answer*: *62.60***

i) Find the P_{30} ***Answer*:*63.80***

j) Find the P_{70} ***Answer*: *72.18***

k) Find the P_{50} ***Answer*: *68***

---

2) The average time for workers to finish a specific task is 38 minutes with a standard deviation 8 minutes. If that data are normally distributed then what percentage of workers finishes the task;

a) Between 30 and 36 minutes ***Answer*: *24.26%***

b) Less than 42 minutes ***Answer*: *69.15%***

c) More than 40 minutes ***Answer*: *40.13%***

d) Within 4 minutes of the mean ***Answer*: *38.3%***

e). Find the time that separates the **fastest 10%** of workers finishing this task.
Note: this is a **cut-off** point and fastest means the bottom 10%

TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 3*** ***then*** ***invNorm (*** $0.10, 38, 8$ ***)*** $\longrightarrow$ ***enter*** **answer: X = 27.74**

f). Find the time that separates the **slowest 15%** of workers finishing this task.
Note: this is a **cut-off** point and **slowest** means the top **15%**

TI-83/84 ***2nd*** $\longrightarrow$ ***DISTR*** $\longrightarrow$ ***Option 3*** ***then*** ***invNorm (*** $0.85, 38, 8$ ***)*** $\longrightarrow$ ***enter*** **answer: X = 46.29**

Using formula to find answers for part e and f

Note:
Also rather using TI-83/84 to find cut-off point, we can use formula $x = \mu + \sigma z$ *and z value* $= -1.28$ *form page 3 of the table for* **bottom 10%** $x = 38 + 8(-1.28) = 27.76$

Note:
Also rather using TI-83/84 to find cut-off point, we can use formula $x = \mu + \sigma z$ *and z value* $= -1.28$ *form page 3 of the table for* **top 15%** $x = 38 + 8(1.0364) = 46.29$

. Find the time that separates the fastest 10% of workers finishing this task. ***Answer*: *27.76***
$x = \mu + \sigma z \Rightarrow x = 38 + 8(-1.28) = 27.76$

. Find the time that separates the slowest 15% of workers finishing this task. ***Answer*: *46.32***
$x = \mu + \sigma z \Rightarrow x = 38 + 8(1.04) = 46.32$

3) The cholesterol level for adult males of a specific racial group is approximately normally distributed with a mean of 4.8 mmol/L and a standard deviation of 0.6 mmol/L.

a) What is the probability that a person has moderate risk if his cholesterol level is more than 1 but less than 2 standard deviations above the mean**:** ***Answer*****: 13.59%**

b) A person has high risk if his cholesterol level is more than 2 standard deviations above the mean. What proportion of the population has high risk ***Answer*****: 2.28%**

c) A person within 1 standard deviation of the mean has normal cholesterol risk What proportion of the population has high risk ***Answer*****: 31.73%**

d) What is the 90th percentile of the distribution (the cholesterol level that exceeds 90% of the population)? ***Answer*****: 5.569**

e) What is the 70th percentile of the distribution, i.e., the cholesterol level that exceeds 70% of the population? ***Answer*****: 5.11**

4). Given the average height of adult male in United States is 65 inches with standard deviation of 8 inches and if the minimum and maximum acceptable heights for being recruited by ARMY is between 55 and 85 inches, then find the percentage of adult male that may be rejected because of their heights? ***Answer*****: *11.19***

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5) The average life of a certain type of motor is 10 years, with a standard deviation of 2 years. Assume that the lives of the motors follow a normal distribution

a) What percentage of motors last longer than 15 years? ***Answer*****: .0062 =.62%**

b) What percentage of motors last less than 7 years? ***Answer*****: 0.668 = 6.68 %**

c) If the manufacturer is willing to replace only 3% of the motors that fail, how long a guarantee should he offer? ***Answer*****: 6.24 years**

d) If the manufacturer is willing to replace only 5% of the motors that fail, how long a guarantee should he offer? ***Answer*****: ? 6.71 years**

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6) A company pays its employees an average wage of $8.25 an hour with a standard deviation of 0.80 cents. If the wages are approximately normally distributed, determine

a. the proportion of the workers getting wages between $6.75 and $10.75 an hour; ***Answer*****: 96%**

b. the minimum wage of the highest 5%. ***Answer*****: $9.57**

c. the minimum wage of the lowest 10%: ***Answer*****: $7.23**

d. What is the 90th percentile of the distribution ***Answer*****: $9.27**

e. What is the 30th percentile of the distribution ***Answer*****: $7.83**

f. What is the 75th percentile of the distribution ***Answer*****: $8.79**

Extra Practice: **Problems F, G 1-10 from practice problem part II on pages 4, 5.**