

For all quizzes in part 3: Be sure you have formula sheet and Table 1 and Table 2.

**Quiz # 9:** This quiz covers all materials on quiz 8 plus estimating population proportions.

What do we estimate? **Population Proportion** ( $P = ?$ )

Know all the new **terminologies** and related **notations** (Point estimate  $\hat{p}$ , Margin of error)

Know all the new **formulas** on **formula sheet** and their related **TI commands**.

To estimate **population proportion** ( $P = ?$ ), know how to use TI (**option A**) or (**formula**  $P = \hat{p} \pm E$ )

Be sure you always have Table 1 as a reference for every estimation problem

**Important:** If confidence level is not given use **95%** as a default.

**Required formula:**  $P = \hat{p} \pm E$        $E = z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$  and **Table 1** (see page 4)

**Example 1:** Use **95%** confidence Level to estimate the percentage of drivers texting while driving when in a sample of 100 drivers 40 text and drive.

First we need to find the point estimate,  $\hat{p} = \frac{x}{n} = \frac{40}{100} = .40$ , **from Table 1 from page 4** for 95% confidence level

the z will be 1.96 so the  $E = z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.96 \sqrt{\frac{.4(1-.4)}{100}} = .096$ , so now  $P = .40 \pm 0.096 = 40\% \pm 9.6\%$

**Final answer:** We have 95% confidence that **between 30.39% to 49.6%** of drivers text while driving.

**Solution by TI 83/84 Calculator**

**stat ----> test----> option**

```
1-PropZInt
x:40
n:100
C-Level:.95
Calculate
```

```
1-PropZInt
(.30398,.49602)
p=.4
n=100
```

**Example 2:** In a sample of 400 applicants for DMV driving test, 280 passed on the first attempt. Find 90% confidence interval of all DMV applicants who pass DMV test on the first attempt.

First  $\hat{p} = \frac{x}{n} = \frac{280}{400} = .70$ , **from Table 1** for 90% confidence level the z will be 1.645 so the

$E = z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.645 \sqrt{\frac{.7(1-.7)}{400}} = .038$  then using  $P = \hat{p} \pm E = .70 \pm 0.038 = 70\% \pm 3.8\%$

**Final answer:** We have 90% confidence that **between 66.2% to 73.8%** of DMV applicant pass driving test on the first attempt.

Estimating Population Proportion $P = \hat{p} \pm E$		
$\hat{p} = \frac{x}{n}$	(Called <b>p-hat</b> is sample proportion and point estimate for population proportion)	$E = \text{Margin of error} \quad E = z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$
Width (difference between upper and lower bounds) = $2E = UB - LB$ so $E = (UB - LB) / 2$		
Point Estimate (middle of upper and lower bounds) = $\hat{p} = (UB + LB) / 2$		
TI-83 <i>stat</i> → <i>test</i> → <i>Option A</i>		

1. If 64% of a sample of 550 people leaving a shopping mall claims to have spent over \$25, determine a 99% confidence interval estimate for the proportion of shopping mall customers who spend over \$25. Interpret your interval.

$$E = 2.5758 \sqrt{\frac{0.64(1-0.64)}{550}} = .0527 \quad P = 0.64 \pm 0.0527 \quad 58.73\% < P < 69.27\%$$

2. In a random sample of machine parts, 18 out of 225 were found to have been damaged in shipment. Establish a 95% confidence interval estimate for the proportion of machine parts that are damaged in shipment. Interpret your interval.

$$\hat{p} = \frac{x}{n} = \frac{18}{225} = 0.08 \quad E = 1.96 \sqrt{\frac{0.08(1-0.08)}{225}} = .0354 \quad P = 0.08 \pm 0.0354 \quad 4.5\% < P < 11.5\%$$

3. A telephone survey of 1000 adults was taken shortly after the U.S. began bombing Iraq. If 832 voiced their support for this action. Create a 99% confidence interval and interpret the interval.

$$\hat{p} = \frac{x}{n} = \frac{832}{1000} = 0.832 \quad E = 2.5758 \sqrt{\frac{0.832(1-0.832)}{1000}} = .0305 \quad P = 0.832 \pm 0.0305 \quad 80.16\% < P < 86.25\%$$

4. An assembly line does a quality check by sampling 50 of its products. It finds that 16% of the parts are defective.

- a. Create a 95% confidence interval for the percent of defective parts for the company and interpret this interval.

$$E = 1.96 \sqrt{\frac{0.16(1-0.16)}{50}} = .102 \quad P = 0.16 \pm 0.102 \quad 0.06\% < P < 26.16\%$$

- b. If we decreased the confidence level to 90% **what would happen to:**

- i. **the critical value?** It decreases from 1.96 to 1.645
- ii. **the margin of error?** It will decrease
- iii. **the confidence interval?** It will become narrower

- c. If the sample size were increased to 200, the same sample proportion were found, and we did a 95% confidence interval; **what would happen to:**

- i. **the critical value?** By just increasing the sample size the critical value will not change
- ii. **the margin of error?** By increasing sample size, the margin of error will decrease.
- iii. **the confidence interval?** By increasing sample size, the interval will be narrower.

5. A nationwide poll was taken of 1400 teenagers (ages 13-18). 630 of them said they have a TV in their room.
- a. Create a 90% confidence interval for the proportion of all teenagers who have a TV in their room and interpret it.

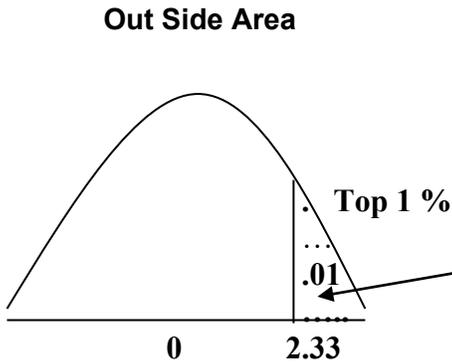
$$\hat{p} = \frac{x}{n} = \frac{630}{1400} = 0.45 \quad E = 1.645 \sqrt{\frac{0.45(1-0.45)}{1400}} = .0133 \quad P = 0.45 \pm 0.0133 \quad 43.67\% < P < 46.33\%$$

What does “90% confidence” mean in this context?

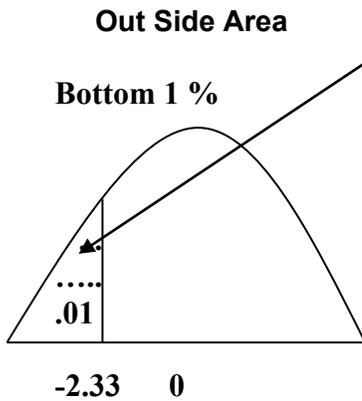
If we increased the confidence level to 99% **what would happen to:**

- i. **the critical value?** It increases 1.645 to 2.5758
  - ii. **the margin of error?** It will increase.
  - iii. **the confidence interval?** It will become wider.
6. If the sample size were changed to 950, the same sample proportion were found, and we did a 90% confidence interval; what would happen to:
- ii. **the critical value?** By just decreasing the sample size the critical value will not change
  - iii. **the margin of error?** It will increase
  - iv. **the confidence interval?** It will become wider.
7. Suppose a 90% confidence interval is stated as (0.3011, 0.4189).
- a. What is the sample proportion from this sample?  $\hat{p} = (UB + LB) / 2 = (0.4189 + 0.3011) / 2 = 0.36$
  - b. What is the margin of error?  $E = (UB - LB) / 2 = (0.4189 - 0.3011) / 2 = 0.0580$

Based on Standard Normal Distribution  $\mu=0$  and  $\sigma=1$



OR

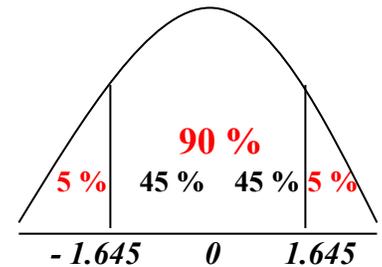


Confidence Level	Out Side Area On left or right Cut-off Point	Z - Value ( ± ) <b>Critical Value</b> = $Z_{\alpha/2}$
99%	.005	± 2.5758
98%	.01	± 2.3263
97%	.015	± 2.1701
96%	.02	± 2.0537
95%	.025	± 1.9600
94%	.03	± 1.8808
92%	.04	± 1.7507
90%	.05	± 1.6450
88%	.06	± 1.5548
86%	.07	± 1.4758
84%	.08	± 1.4051
82%	.09	± 1.3408
80%	.10	± 1.2816
78%	.11	± 1.2265
76%	.12	± 1.1750
70%	.15	± 1.0364
60%	.20	± 0.8416
50%	.25	± 0.6749
40%	.30	± 0.5244

**How to find the Z -value for confidence intervals.**

**Example: Find the Z - value for 90% confidence interval**

1. Divide 90% = 0.90 by 2,  $\Rightarrow .90 / 2 = 0.45$
2. Subtract 0.45 from 0.5  $\Rightarrow .5 - 0.45 = .05$
3. Look for area close to 0.05 from **inside** the table (page1).
- 4 **Find its corresponding Z-value (- 1.645)**



TI-83/84 2nd  $\rightarrow$  Distr  $\rightarrow$  Option 3 **input** (% , 0 , 1)

**Example:** 2nd  $\rightarrow$  Distr  $\rightarrow$  Option 3 **input** (.05 , 0 , 1) enter , then the answer will be - 1.645

**Example:** 2nd  $\rightarrow$  Distr  $\rightarrow$  Option 3 **input** (.95 , 0 , 1) enter , then the answer will be 1.645

**Hint for TI** % is the area to the left of the cut off point.