HW #8 Solution

7.37 **a** $p=.3;\ SE(\hat{p})=\sqrt{\frac{pq}{n}}=\sqrt{\frac{.3(.7)}{100}}=.0458$

b $p=.1;\ SE(\hat{p})=\sqrt{\frac{pq}{n}}=\sqrt{\frac{.1(.9)}{400}}=.015$

c $p=.6;\ SE(\hat{p})=\sqrt{\frac{pq}{n}}=\sqrt{\frac{.6(.4)}{250}}=.0310$

7.38 For each of the three binomial distributions, calculate *np* and *nq*:
a *np* = 2.5 and *nq* = 47.5 **b** *np* = 7.5 and *nq* = 67.5 **c** *np* = 247.5 and *nq* = 2.5
The normal approximation to the binomial distribution is only appropriate for part **b**, when *n* = 75 and *p* = .1.

7.39 **a** Since $\hat{p}$ is approximately normal, with standard deviation $SE(\hat{p})=\sqrt{\frac{pq}{n}}=\sqrt{\frac{.4(.6)}{75}}=.0566$, the probability of interest is

$$P(\hat{p}\le .43)=P(z\le\frac{.43-.4}{.0566})=P(z\le .53)=.7019$$

b The probability is approximated as

$$P(.35\le\hat{p}\le .43)=P\left[\frac{.35-.4}{.0566}\le z\le\frac{.43-.4}{.0566}\right]$$
$$=P(-.88\le z\le .53)=.7019-.1894=.5125$$

7.43 **a** For $n=100$ and $p=.19$, $np=19$ and $nq=81$ are both greater than 5. Therefore, the normal approximation will be appropriate, with mean $p=.19$ and $SE=\sqrt{\frac{pq}{n}}=\sqrt{\frac{.19(.81)}{100}}=.03923$.

b $P(\hat{p}>.25)=P\left(z>\frac{.25-.19}{.03923}\right)=P(z>1.53)=1-.9370=.0360$

c $P(.25<\hat{p}<.30)=P\left(\frac{.25-.19}{.03923}<z<\frac{.30-.19}{.03923}\right)=P(1.53<z<2.80)=.9974-.9370=.0604$

d The value $\hat{p}=.30$ lies

$$z=\frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}}=\frac{.30-.19}{.03923}=2.80$$

standard deviations from the mean. Also, $P(\hat{p}\ge .30)=P(z\ge 2.80)=1-.9974=.0026$. This is an unlikely occurrence, assuming that $p=.19$. Perhaps the sampling was not random, or the 19% figure is not correct.

7.47 **a** The random variable $\hat{p}$, the sample proportion of consumers who like nuts or caramel in their chocolate, has a binomial distribution with $n=200$ and $p=.75$. Since $np=150$ and $nq=50$ are both greater than 5, this binomial distribution can be approximated by a normal distribution with mean $p=.75$ and $SE=\sqrt{\frac{.75(.25)}{200}}=.03062$

b $P(\hat{p}>.80)=P\left(z>\frac{.80-.75}{.03062}\right)=P(z>1.63)=1-.9484=.0516$

c From the Empirical Rule (and the general properties of the normal distribution), approximately 95% of the measurements will lie within 2 (or 1.96) standard deviations of the mean:

$$p\pm 2SE \quad\Rightarrow\quad .75\pm 2(.03062)$$
$$.75\pm .06 \quad \text{or} \quad .69 \text{ to } .81$$

8.3 For the estimate of μ given as $\bar{x}$, the margin of error is $1.96\ SE=1.96\frac{\sigma}{\sqrt{n}}$.

a $1.96\sqrt{\frac{0.2}{30}}=.160$ **b** $1.96\sqrt{\frac{0.9}{30}}=.339$ **c** $1.96\sqrt{\frac{1.5}{30}}=.438$

8.4 Refer to Exercise 8.3. As the population variance σ^2 increases, the margin of error also increases.

8.5 The margin of error is $1.96\ SE = 1.96\frac{\sigma}{\sqrt{n}}$, where σ can be estimated by the sample standard deviation *s* for large values of *n*.

a $1.96\sqrt{\frac{4}{50}} = .554$ **b** $1.96\sqrt{\frac{4}{500}} = .175$ **c** $1.96\sqrt{\frac{4}{5000}} = .055$

8.6 Refer to Exercise 8.5. As the sample size *n* increases, the margin of error decreases.

8.17 **a** The point estimate for *p* is given as $\hat{p} = \frac{x}{n} = .51$ and the margin of error is approximately

$$1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{.51(.49)}{900}} = .0327$$

b The sampling error was reported by using the maximum margin of error using $p = .5$, and by rounding off to the nearest percent:

$$1.96\sqrt{\frac{\hat{p}\hat{q}}{n}} = 1.96\sqrt{\frac{.5(.5)}{900}} = .0327 \text{ or } \pm 3\%$$

8.21 A point estimate for the mean length of time is $\bar{x} = 19.3$, with margin of error

$$1.96\ SE = 1.96\frac{\sigma}{\sqrt{n}} \approx 1.96\frac{s}{\sqrt{n}} = 1.96\frac{5.2}{\sqrt{30}} = 1.86$$

8.23 Similar to Exercise 8.22, with a 90% confidence interval for μ given as

$$\bar{x} \pm 1.645\frac{\sigma}{\sqrt{n}}$$

where σ can be estimated by the sample standard deviation *s* for large values of *n*.

a $.84 \pm 1.645\sqrt{\frac{.086}{125}} = .84 \pm .043$ or $.797 < \mu < .883$

b $21.9 \pm 1.645\sqrt{\frac{3.44}{50}} = 21.9 \pm .431$ or $21.469 < \mu < 22.331$

c Intervals constructed in this manner will enclose the true value of μ 90% of the time in repeated sampling. Hence, we are fairly confident that these particular intervals will enclose μ.

8.36 **a** The time to complete an online order is probably not mound-shaped. The minimum value of *x* is zero, and there is an average time of $\mu = 4.5$, with a standard deviation of $\sigma = 2.7$. If we calculate $\mu - 2\sigma = -.9$, leaving no possibility for a measurement to fall more than two standard deviations below the mean. For a mound-shaped distribution, approximately 2.5% should fall in that range. The distribution is probably skewed to the right.

b Since *n* is large, the Central Limit Theorem ensures that the sample mean $\bar{x}$ is approximately normal, and the standard normal distribution can be used to construct a confidence interval for μ.

c The 95% confidence interval for μ is

$$\bar{x} \pm 1.96\frac{s}{\sqrt{n}} = 4.5 \pm 1.96\frac{2.7}{\sqrt{50}} = 4.5 \pm .478 \text{ or } 4.022 < \mu < 4.978$$

8.37 **a** The 99% confidence interval for μ is

$$\bar{x} \pm 2.58\frac{s}{\sqrt{n}} = 98.25 \pm 2.58\frac{0.73}{\sqrt{130}} = 98.25 \pm .165 \text{ or } 98.085 < \mu < 98.415$$

b Since the possible values for μ given in the confidence interval does not include the value μ = 98.6, it is not likely that the true average body temperature for healthy humans is 98.6, the usual average temperature cited by physicians and others.