Statistics 1, Sections 4 and 6
Fall 2008
Solution to HW #5

5.5 **a** $C_2^8(.3)^2(.7)^6 = \frac{8(7)}{2(1)}(.09)(.117649) = .2965$

b $C_0^4(.05)^0(.95)^4 = (.95)^4 = .8145$

c $C_3^{10}(.5)^3(.5)^7 = \frac{10(9)(8)}{3(2)(1)}(.5)^{10} = .1172$

d $C_1^7(.2)^1(.8)^6 = 7(.2)(.8)^6 = .3670$

5.7 **a** For $n = 7$ and $p = .3$, $P(x = 4) = C_4^7(.3)^4(.7)^3 = .097$.

b These probabilities can be found individually using the binomial formula, or alternatively using the cumulative binomial tables in Appendix I.

$$P(x \le 1) = p(0) + p(1)$$
$$= C_0^7(.3)^0(.7)^7 + C_1^7(.3)^1(.7)^6$$
$$= (.7)^7 + 7(.3)(.7)^6 = .08235 + .24706 = .329$$

or directly from the binomial tables in the row marked $k = 1$.

c Refer to part **b**. $P(x > 1) = 1 - P(x \le 1) = 1 - .329 = .671$.

d $\mu = np = 7(.3) = 2.1$

e $\sigma = \sqrt{npq} = \sqrt{7(.3)(.7)} = \sqrt{1.47} = 1.212$

5.10 Refer to Exercise 5.9. If $p = .5$, $p(x) = C_x^n(.5)^x(.5)^{n-x} = C_x^n(.5)^n$. Since $C_x^n = C_{n-x}^n$ (from Exercise 5.9), you can see that for any value of *k*,

$$P(x = k) = C_k^n(.5)^n = C_{n-k}^n(.5)^n = P(x = n - k)$$

This indicates that the probability distribution is exactly symmetric around the center point .5*n*.

5.20 Although there are $n = 30$ days on which it either rains (S) or does not rain (F), the random variable *x* would not be a binomial random variable because the trials (days) are *not independent*. If there is rain on one day, it will probably affect the probability that there will be rain on the next day.

5.21 Although there are trials (telephone calls) which result in either a person who will answer (S) or a person who will not (F), the number of trials, *n*, is not fixed in advance. Instead of recording *x*, the number of *successes* in *n* trials, you record *x*, the number of *trials* until the first success. This is *not* a binomial experiment.

5.23 Define *x* to be the number of alarm systems that are triggered. Then $p = P[\text{alarm is triggered}] = .99$ and $n = 9$. Since there is a table available in Appendix I for $n = 9$ and $p = .99$, you should use it rather than the binomial formula to calculate the necessary probabilities.

a $P[\text{at least one alarm is triggered}] = P(x \ge 1) = 1 - P(x = 0) = 1 - .000 = 1.000$.

b $P[\text{more than seven}] = P(x > 7) = 1 - P(x \le 7) = 1 - .003 = .997$

c $P[\text{eight or fewer}] = P(x \le 8) = .086$

5.24 Define *x* to be the number of people with Rh-negative blood. Then p=P(Rh-negative)=1-0.85=0.15 and $n = 2$. Since there is no table available for p=0.15 you will have to calculate the probabilities using the binomial formula.

P(both Rh-negative)=P(x=2)=C_2^2 $(0.15)^2(0.85)^0$=0.0225

5.29 Define *x* to be the number of fields infested with whitefly. Then $p = P[\text{infected field}] = .1$ and $n = 100$.

a $\mu = np = 100(.1) = 10$

b Since n is large, this binomial distribution should be fairly mound-shaped, even though $p = .1$. Hence you would expect approximately 95% of the measurements to lie within two standard deviation of the mean with $\sigma = \sqrt{npq} = \sqrt{100(.1)(.9)} = 3$. The limits are calculated as

$$\mu \pm 2\sigma \Rightarrow 10 \pm 6 \text{ or from 4 to 16}$$

c From part **b**, a value of $x = 25$ would be very unlikely, assuming that the characteristics of the binomial experiment are met and that $p = .1$. If this value were actually observed, it might be possible that the trials (fields) are not independent. This could easily be the case, since an infestation in one field might quickly spread to a neighboring field. This is evidence of *contagion*.

5.33 Define x to be the number of Americans who are "tasters". Then, $n = 20$ and $p = .7$. Using the binomial tables in Appendix I,

a $P(x \geq 17) = 1 - P(x \leq 16) = 1 - .893 = .107$

b $P(x \leq 15) = .762$