Solutions to Exam 2 Practice Problems

1. a) $P(X=0) = 1-(P(1)+P(2)+\cdots+P(4))$
$= 1-(0.28+0.2+0.1+0.02)$
$= 1-0.6 = \boxed{0.40}$

b) $\mu = \sum x \cdot p(x) = 0 \cdot p(0) + 1 \cdot p(1) + \cdots + 4 \cdot p(4)$
$= 0(.40) + 1(0.28) + 2(0.20) + 3(0.10) + 4(0.02)$
$= 0 + 0.28 + 0.40 + 0.30 + 0.08$
$\boxed{\mu = 1.06}$

$\sigma^2 = \sum (x-\mu)^2 \cdot p(x) = (0-1.06)^2 (0.40) + (1-1.06)^2 (0.28)$
$+ (2-1.06)^2 (0.20) + (3-1.06)^2 (0.10) + (4-1.06)^2 (0.02)$
$= 0.44944 + 0.001008 + 0.17672 + 0.37636 + 0.172872$
$= 1.1764$
$\sigma = \sqrt{\sigma^2} = \sqrt{1.1764} = 1.08462$ $\boxed{\sigma \approx 1.08}$

c) Over many, many days, Dr. Seuss receives an average of 1.06, or about 1, emergency call per day.

2. X = # of defective bolts in the 10 selected
X is binomial, $n=10$, $p=0.15$

a) $P(X=3) = C^{10}_{3}\ 0.15^3 (1-0.15)^{10-3}$
$= C^{10}_{3}\ 0.15^3 (0.85)^7$
$= 120 (0.15^3)(0.85^7) = 0.130$

b) $P(X \geq 2) = 1 - P(X=0) - P(X=1)$
$= 1 - C^{10}_{0}(.15^0)(.85^{10}) - C^{10}_{1}(.15^1)(.85^9)$
$= 1 - 0.1969 - 0.3474$
$= 0.456$

c) $\mu = np = 10(.15) = 1.5$
$\sigma = \sqrt{npq} = \sqrt{10(.15)(.85)} = 1.129$

2d) $\mu = np = 200(.15) = 30$

$\sigma = \sqrt{npq} = \sqrt{200(.15)(.85)} = 5.05$

since $np = 30 > 5$ and $nq > 5$, the distribution will be approx. mound-shaped, the Empirical Rule say about 99.7% of values of the RV will be in

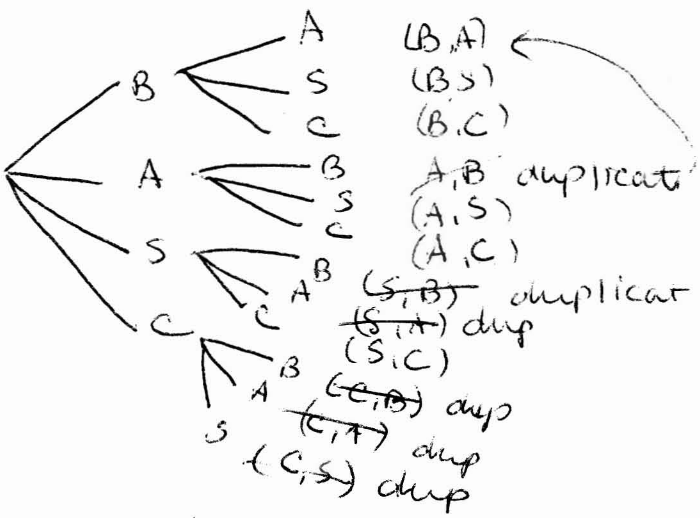
$\mu \pm 3\sigma$

$30 \pm 3(5.05) = 30 \pm 15.15$

$(14.85, 45.15)$

So $X = 50$ is <u>very</u> unusual!

3. a) $S = \{(B,A), (B,S), (B,C), (A,S), (A,C), (S,C)\}$



b)

X	P(X)
0	1/6
1	4/6
2	1/6

outcome	X
B,A	2
B,S	1
B,C	1
A,S	1
A,C	1
S,C	0

c) $P(X=2 \mid X \geq 1) = \dfrac{P(X=2 \text{ and } X \geq 1)}{P(X \geq 1)}$

$= \dfrac{P(X=2)}{P(X \geq 1)} = \dfrac{1/6}{5/6} = 1/5$

4. a) NOT binomial
   b) NOT binomial
   c) Binomial
   d) NOT binomial
   e) Binom.
   f) NOT binomial

5) $P(F) = 0.60$ $\quad P(D) = 0.40$

$P(Y|F) = 0.10$ $\quad P(Y|D) = 0.20$

a) "Hypothetical 100 method"

	Yellow	Not Yellow	Total
Foreign	$0.1(60) = 6$	54	60 ← second, 60% of cars are foreign
Domestic	$0.2(40) = 8$	32	40
Total	14	86	100 ← Start by assuming there are 100 cars in total

$P(\text{Yellow}) = \frac{\#\text{ Yellow cars}}{\text{Total }\#\text{ Cars}} = \frac{14}{100} = \boxed{0.14}$

Alternate Solution: or $P(\text{Yellow}) = P(F)\cdot P(Y|F) + P(D)\cdot P(Y|D)$

$= 0.6(0.1) + 0.4(0.2)$

$= 0.06 + 0.08 = 0.14$

b) $P(D|Y) = \frac{P(D \cap Y)}{P(Y)} = \frac{8}{14} = \frac{4}{7}$ ← from table

Alternate Soln. -OR- $P(D|Y) = \frac{P(D)\cdot P(Y|D)}{P(Y)}$

$= \frac{0.4(0.2)}{0.14} = \frac{0.08}{0.14} = \frac{4}{7}$

6. a)

winnings, one play of the game

X	$p(X)$
\$2	$1/6$
\$4	$1/6$
\$6	$1/6$
−\$6	$1/2$

b) $E(X) = \sum_i x \cdot p(x)$

$= 2(1/6) + 4(1/6) + 6(1/6) + 1/2(-6)$

$= 1/3 + 2/3 + 1 - 3$

$= 2 - 3 = -1$

c) if you play the game many, many times, you will lose an average of \$1 per game.

7. a) X = weight of newborn

$$P(6.0 < X < 8.0) = P\left(\frac{6.0-7.2}{1.1} < Z < \frac{8.0-7.2}{1.1}\right)$$

$= P(-1.09 < Z < 0.73)$

$= P(Z < 0.73) - P(Z < -1.09)$

$= 0.7673 - 0.1379$

$= \boxed{0.6291}$

b) $P(X > 9.5) = P\left(Z > \frac{9.5-7.2}{1.1}\right)$

$= P(Z > 2.09)$

$= 1 - P(Z \leq 2.09)$

$= 1 - 0.9817 = \boxed{0.0183}$

c) $\mu \pm 3\sigma$ by Empirical Rule

$7.2 \pm 3(1.1)$

7.2 ± 3.3

$\boxed{(3.9, 10.5)}$