Statistics 1, Section 4 **Name** Solutions

Exam 2 **October 31, 2008 (Happy Halloween!)**

Calculators and one 8.5" by 11" sheet of handwritten notes allowed. Show all work and answers clearly in the space provided. Partial credit can only be determined based on the work shown on this exam paper. Tables are included at the end of the exam. Each question is worth 20 points.

1. **A bowl contains 4 black balls and 2 white balls. Two balls are randomly selected *without* replacement. Let the random variable X represent the number of black balls among the two drawn.**
   a. **What are the possible values of X?** 0, 1, 2
   b. **Give the probability distribution of X.**

x	p(x)
0	2/30
1	16/30
2	12/30

white on first draw

$$P(X=0)=P(W_1W_2)=P(W_1)\cdot P(W_2|W_1)$$
$$=\frac{2}{6}\cdot\frac{1}{5}=\frac{2}{30}$$

$$P(X=2)=P(B_1B_2)=\frac{4}{6}\cdot\frac{3}{5}=\frac{12}{30}$$

$$P(X=1)=1-P(0)-P(2)=1-\frac{2}{30}-\frac{12}{30}$$
$$=\frac{16}{30}$$

   c. **Calculate the mean and standard deviation of the probability distribution in part (a).**

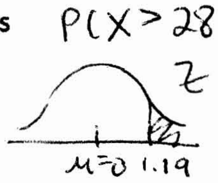
$$\mu=EX=\sum x\cdot p(x)=0(2/30)+1(16/30)+2(12/30)$$
$$=0+\frac{16+24}{30}=\frac{40}{30}=4/3$$

$$\sigma^2=\sum(x-\mu)^2\cdot p(x)=(0-4/3)^2(2/30)+(1-\tfrac{4}{3})^2(16/30)+(2-\tfrac{4}{3})^2(12/30)$$
$$=\frac{16}{9}\left(\frac{2}{30}\right)+\frac{1}{9}\left(\frac{16}{30}\right)+\left(\frac{2}{3}\right)^2\left(\frac{12}{30}\right)=\frac{96}{270}\qquad \sigma=\sqrt{\frac{96}{270}}=\boxed{0.60}$$

2. **The lengths of human pregnancies are normally distributed with mean 266 days and standard deviation 16 days. What is the probability that a randomly selected pregnancy lasts**

   a. **Less than 270 days** $P(X<270)=P\left(Z<\frac{270-266}{16}\right)=P(Z<0.25)$
$$=0.5987$$

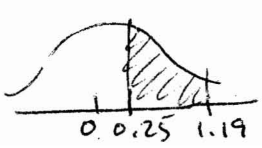
   b. **Over 285 days** $P(X>285)=P\left(Z>\frac{285-266}{16}\right)=P(Z>1.19)$



$$=1-P(Z\le 1.19)=1-.8830=0.117$$

   c. **Between 270 and 285 days**

$$P(270<X<285)=P(0.25<Z<1.19)$$
$$=P(Z\le 1.19)-P(Z\le 0.25)$$
$$=.8830-.5987=0.2843$$



d. Give an interval centered on the mean which contains 95% of all lengths of human pregnancies.

$\mu \pm 2\sigma = 266 \pm 2(16)$

$266 \pm 32 \quad (234, 298)$

3. 98% of the XYZ Airlines' flights arrive at their destination on time. If ten flights are randomly selected, what is the probability that Binomial $n=10$ $p=0.98$

a. Exactly 9 are on time

$P(X=9) = C_9^{10}(0.98^9)(0.02)^{10-9} = 10(.98^9)(.02)$

$= 0.167$

b. At most 8 are on time

$P(X \le 8) = 1 - P(X \ge 9) = 1 - P(9) - P(10)$

$= 1 - 0.167 - C_{10}^{10} 0.98^{10} (0.02)^0$

$= 1 - .167 - 0.817 = 0.016$

4. At ACME Explosives, a corporation whose CEO is the infamous Bugs Bunny, 65% of the employees are males while 35% are females. A survey on employee satisfaction reveals that, of the males, 50% are satisfied with their jobs. Of the females, 60% are satisfied with their jobs. If an employee is randomly selected from ALL employees of ACME Explosives, what is the probability that

S = employee satisfied w/ job

	S	S^c	Total
M	325	325	650
F	210	140	350
Total	535	465	1000

a. The employee is satisfied with his/her job

$P(S) = \frac{535}{1000} = 0.535$

b. Given the employee is satisfied with his/her job, what is the probability the employee is female

$P(F|S) = \frac{210}{535} = 0.393$

5. A fair coin is tossed 100 times and the number of heads is counted.

a. How many outcomes contain exactly 1 heads? 99 heads?

$C_1^{100} = 100$ contain 1 heads $\qquad C_{99}^{100} = 100$ contain 99 heads

b. How many outcomes contain exactly 2 heads? 98 heads?

$C_2^{100} = 4950$ have 2 heads $\qquad C_{98}^{100} = 4950$ have 2 heads

c. Explain why the number of outcomes having exactly k heads is always the same as the number having 100-k heads. Either show using mathematical formulas or give a convincing verbal argument. (Hint: see parts (a) and (b).)

$$C_k^{100} = \frac{100!}{k!(100-k)!} \qquad C_{100-k}^{100} = \frac{100!}{(100-k)!(100-[100-k])!} = \frac{100!}{(100-k)!\,k!}$$

$$C_k^{100} = C_{100-k}^{100}$$

d. Let X = the number of heads in the 100 tosses, is the following calculation correct? Why or why not?

$$P(X = 2) = P(SSFF \cdots F) = \left(\frac{1}{2}\cdot\frac{1}{2}\right)\cdot\left(\frac{1}{2}\cdot\frac{1}{2}\cdots\frac{1}{2}\right) = \left(\frac{1}{2}\right)^{100}$$

not correct, need to multiply by C_2^{100} to account for all possible orderings of 2 successes in 100 trials

e. What is the mean and standard deviation of X?

$\mu = E(X) = np = 100(1/2) = 50$

$\sigma^2 = npq = 100(1/2)(1/2) = 25 \qquad \sigma = \sqrt{25} = 5$

f. Give an interval where 95% of the values of X will fall. Would it be unusual to observe 65 heads?

$np > 5 \quad nq > 5$ so distn. mound-shaped

Empirical Rule

$\mu \pm 2\sigma$

$50 \pm 2(5)$

50 ± 10

$(40, 60)$ it would be unusual to observe 65 heads since it does not fall in this interval