Solutions to Practice Problems for Exam 3
Stat 1, Fall 2008

1. a) This is most likely an observational study. If it were a designed experiment, the kids would have to be randomly assigned to daycare (the treatment) or not. Most parents would not permit this decision to be made based on random chance.

   b) An observational study cannot prove cause and effect. It can only show that variables are correlated. A designed experiment is needed to prove cause and effect.

2. Randomization, Placebo and Double-blinding. See slides for Section 7.2 for definitions.

3. $\mu = 40, \ \sigma = 6$

   a) $n = 36$, $\quad P(37 < \bar{X} < 42) = P\left(\frac{37-\mu_{\bar{x}}}{\sigma_{\bar{x}}} < Z < \frac{42-\mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$

$$= P\left(\frac{37-\mu}{\sigma/\sqrt{n}} < \bar{X} < \frac{42-\mu}{\sigma/\sqrt{n}}\right)$$

$$= P\left(\frac{37-40}{6/\sqrt{36}} < Z < \frac{42-40}{6/\sqrt{36}}\right)$$

$$= P(-3 < Z < 2)$$

$$= 0.9772 - 0.0013 = \boxed{0.9759}$$

3b) T = total weight of 900 boxes

$P(T > 36{,}450) = P\left(\frac{T}{900} > \frac{36{,}450}{900}\right)$

$= P(\bar{X} > 40.5)$

$= P\left(\frac{\bar{X} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} > \frac{40.5 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$

$= P\left(Z > \frac{40.5 - 40}{6/\sqrt{900}}\right)$

$= P(Z > 2.50)$

$= 1 - 0.9938 = 0.0062$

c) No. since the sample size's are large ($n \geq 30$), $\bar{X}$ is approximately normal even if the distribution of box weights is NOT normal.

4. $\mu = 600$, $\sigma = 100$

a) X = score of randomly selected subject

$P(500 < X < 735) = P\left(\frac{500-\mu}{\sigma} < Z < \frac{735-\mu}{\sigma}\right)$

$= P\left(\frac{500-600}{100} < Z < \frac{735-600}{100}\right)$

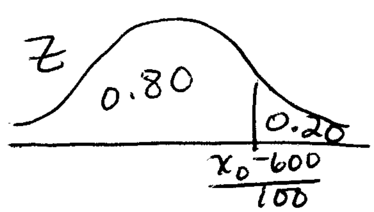
$= P(-1.00 < Z < 1.35)$

$= 0.9115 - 0.1587 = 0.7528$

b) $P(X > x_0) = 0.20$ (x_0 = lowest acceptable score)

$P\left(\frac{X-\mu}{\sigma} > \frac{x_0 - \mu}{\sigma}\right) = 0.20$

$P\left(Z > \frac{x_0 - 600}{100}\right) = 0.20$



4b) using Table III, we look up 0.80 in the body, the z-value is 0.84.

Thus,
$$0.84 = \frac{x_0 - 600}{100}$$

$$100(0.84) = x_0 - 600$$
$$100(0.84) + 600 = x_0$$
$$\boxed{684 = x_0}$$

c) $n=50$, $P(\bar{x} > 615) = P\left(\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} > \frac{615 - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \right)$

$$= P\left(Z > \frac{615 - \mu_{\bar{x}}}{\sigma_{\bar{x}}}\right)$$
$$= P\left(Z > \frac{615 - \mu}{\sigma/\sqrt{n}}\right) = P\left(Z > \frac{615 - 600}{100/\sqrt{50}}\right)$$
$$= P(Z > 1.06) = 1 - 0.8554$$
$$= 0.1446$$

5. Technically, we need $n \geq 30$ here, but i'll go ahead & use the large-sample confidence interval.

a) $\bar{x} \pm z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ $\alpha = 0.10$, $z_{\alpha/2} = z_{0.05}$

$$72.19 \pm 1.645\left(\frac{2.995}{\sqrt{8}}\right)$$
$$72.19 \pm 1.74$$
$$(70.45, 73.93)$$

b) we need to assume population is normally distributed. (actually, need large-sample size here.)

5c) MoE = 1.74

d) No, about 10% of the time μ will fall OUTSIDE the interval (10% = 1 − 90%)
conf. level

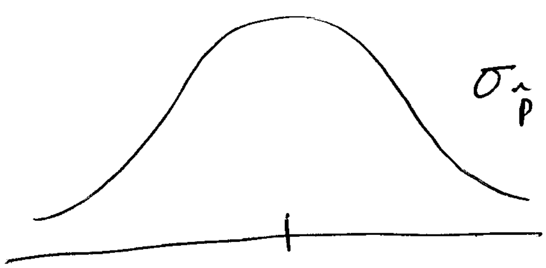
e) wider

f) MoE determines width.

$$MoE = Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$
increasing n, will decrease the MoE so it will decrease the width of the conf. int.

6. $p = 0.52$ $n = 1000$

since $np = 1000(0.52) = 520 > 5$
$nq = 1000(0.48) = 480 > 5$
$\hat{p}$ is normally distributed



$\mu_{\hat{p}} = p = 0.52$

$$\sigma_{\hat{p}} = \sqrt{\frac{pq}{n}} = \sqrt{\frac{0.52(0.48)}{1000}} = 0.0158$$

b) i) $P(0.49 < \hat{p} < 0.53) = P\left(\frac{0.49-0.52}{0.0158} < Z < \frac{0.53-0.52}{0.0158} \right)$

$= P(-1.90 < Z < 0.63)$

$= 0.7357 - 0.0287$

$= 0.7070$

6b) ii) $P(\hat{p} > 0.57) = P\left(Z > \frac{0.57 - 0.52}{0.0158}\right)$

$= P(Z > 3.16)$

$= 1 - 0.9992$

$= 0.0008$

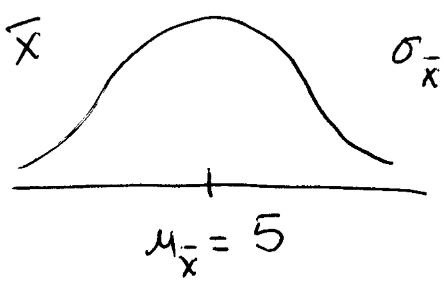
c) $\mu_{\hat{p}} \pm 2\sigma_{\hat{p}}$ by Empirical Rule

$0.52 \pm 2(0.0158)$

0.52 ± 0.0316

$(0.49, 0.55)$

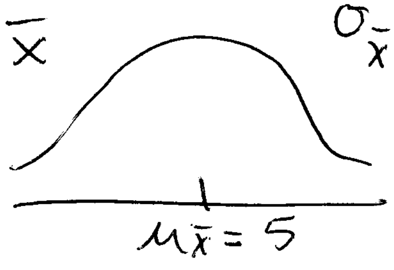
7. a) $n \geq 30$ so $\bar{X}$'s normal



$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{64}} = \frac{1}{8}$

b) n is small, population is not known to be normal, $\Rightarrow$ shape of distn. of $\bar{X}$ is UNKNOWN. $\underbrace{\mu_{\bar{X}} = \mu = 5, \ \sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{16}} = \frac{1}{4}}_{\text{regardless of } n}$

c) since pop. normal $\Rightarrow$ $\bar{X}$ normal for any n



$\sigma_{\bar{X}} = \frac{\sigma}{\sqrt{n}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$