Statistics 1/Section 4
Exam 3

Name Solutions
November 26, 2008

Calculators and one 8.5" by 11" sheet of handwritten notes allowed. Show all work and answers clearly in the space provided. Partial credit can only be determined based on the work shown on this exam paper. Tables are included at the end of the exam.

1) Identify each of the following as an observational study or designed experiment.

a) In a recent study, a sample of 1000 Americans were asked if they ate out at least once in the previous week. 77% indicated that they had eaten out at least once.

observation study

b) A study is conducted to compare the job satisfaction between employees in management and non-management positions. A random sample of employees indicates that 58% of managers are satisfied with their job whereas 75% of non-managers were satisfied with their job.

observational study

c) A study is conducted to determine if a new sleeping pill is effective. 25 people are randomly assigned to take the sleeping pill while the other 25 are given a placebo. The average amount of time it takes to go to sleep is determined for each group and compared.

designed experiment

d) Which of the studies in parts (a) through (c) would it be possible to establish cause and effect under?

(c) only.
Cause & effect can only be established by a designed experiment

2) A study is conducted to estimate the mean square footage of a home in Newville. A random sample of 117 homes gives a mean of 1654 sq. ft. and a standard deviation of 524 sq. feet.

a) Calculate a 95% confidence interval for the mean square footage.

$\bar{X} \pm Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ $\quad Z_{\alpha/2} = Z_{0.05/2} = Z_{0.025} = 1.96$

$1654 \pm 1.96\left(\frac{524}{\sqrt{117}}\right)$

1654 ± 94.9

$(1559.1, 1748.9)$

b) Calculate a 99% confidence interval for the mean square footage.

$1654 \pm 2.58\left(\frac{524}{\sqrt{117}}\right)$ $\quad Z_{\alpha/2} = Z_{0.01/2} = Z_{0.005} = 2.58$

look up .995 in body of table

1654 ± 124.99

$(1529.01, 1778.99)$

c) If the sample size increases, what happens to the width of the confidence interval?

the width of the confidence interval decreases as sample size, n, increases

width $= 2Z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ ← since n is in the denominator larger n causes the width to get smaller

3) Suppose the number of widgets produced in a day by an employee at Widgets R Us is a random variable with a mean of 20.4 and a standard deviation of 3.1. If 65 employees report to work today,

17 a. what is the approximate probability the ***total number*** of widgets they produce is less than 1300?

$\mu = 20.4 \qquad \sigma = 3.1 \qquad n = 65$

$$P(T < 1300) = P\left(\frac{T}{65} < \frac{1300}{65}\right) = P(\bar{X} < 20)$$

$$= P\left(\frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}} < \frac{20 - \mu_{\bar{X}}}{\sigma_{\bar{X}}}\right)$$

$$= P\left(Z < \frac{20 - \mu}{\sigma/\sqrt{n}}\right)$$

$$= P\left(Z < \frac{20 - 20.4}{3.1/\sqrt{65}}\right)$$

$$= P\left(Z < \cancel{0.38} \; \frac{20 - 20.4}{0.385}\right)$$

$$= P(Z < -1.04) = \boxed{0.1492}$$

3 b. Do we need to assume the population is normally distributed to do part (a)?

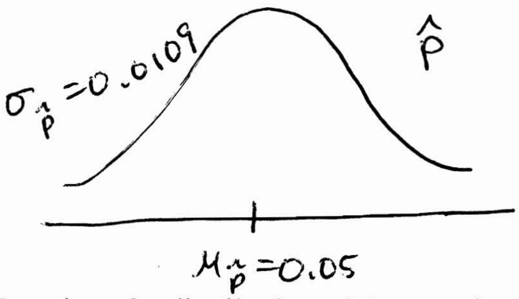
No. The sample size is large ($n = 65$) so the $\bar{X}$'s will be normally distributed even if the population of # of widgets in a day by an employee ~~# by an employee~~ produced is not normally distributed.

4) Suppose a machine produces bolts. The percent of defective bolts is 5%. Suppose a simple random sample of 400 bolts is taken.

a) What is the distribution of the sample proportion of defectives? (Include information about the shape, center and spread.)

$np = 400(.05) = 20 > 5 \quad nq = 400(.95) = 380 > 5$

so $\hat{p}$ ~~the~~ is normally distributed



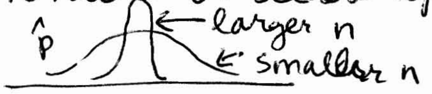
$\mu_{\hat{p}} = p = 0.05$

$\sigma_{\hat{p}} = \sqrt{\dfrac{pq}{n}}$

$= \sqrt{\dfrac{0.05(0.95)}{400}} = 0.0109$

b) How does the distribution of the sample proportion changes as the sample size increases?

the distribution gets skinnier or less spread out about the mean.



c) What is the probability the sample contains 7% or more defectives?

$P(\hat{p} \geq 0.07) = P\left(\dfrac{\hat{p} - \mu_{\hat{p}}}{\sigma_{\hat{p}}} \geq \dfrac{0.07 - \mu_{\hat{p}}}{\sigma_{\hat{p}}}\right)$

$= P\left(Z \geq \dfrac{0.07 - 0.05}{0.0109}\right)$

$= P(Z \geq 1.83)$

$= 1 - P(Z < 1.83)$

$= 1 - 0.9664 = \boxed{0.0336}$

5) SAT verbal scores are normally distributed with mean 430 and standard deviation 120.

a) What is the probability a randomly selected student's score is over 600? X = r.s. student's score

$$P(X > 600) = P\left(\frac{X-\mu}{\sigma} > \frac{600-\mu}{\sigma}\right)$$

$$= P\left(Z > \frac{600-430}{120}\right)$$

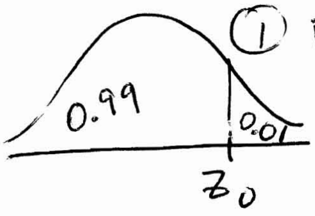
$$= P(Z > 1.42)$$

$$= 1 - P(Z \le 1.42) = 1 - 0.9222 = 0.0778$$

b) What is the minimum score a student must attain to be in the top 1% of all SAT verbal scores?

x_0 = min. score to be in top 1%.

$$P(X > x_0) = 0.01$$



① Find corresponding z-score, called z_0

$z_0 = 2.33$

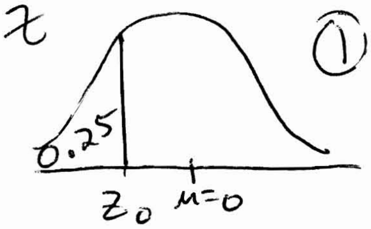
② convert to X scale

$$\frac{x_0 - \mu}{\sigma} = 2.33 \Rightarrow \frac{x_0 - 430}{120} = 2.33$$

$$\Rightarrow x_0 = 430 + 120(2.33)$$

$$\boxed{= 709.6}$$

c) Give the first quartile of SAT scores.



① Find z-score, z_0, with area 0.25 below. Look up 0.25 in BODY of table.

$z_0 = -0.67$

② convert to X scale

$$\frac{x_0 - \mu}{\sigma} = -0.67$$

$$\frac{x_0 - 430}{120} = -0.67$$

$$x_0 = 430 + 120(-0.67) = \boxed{349.6}$$