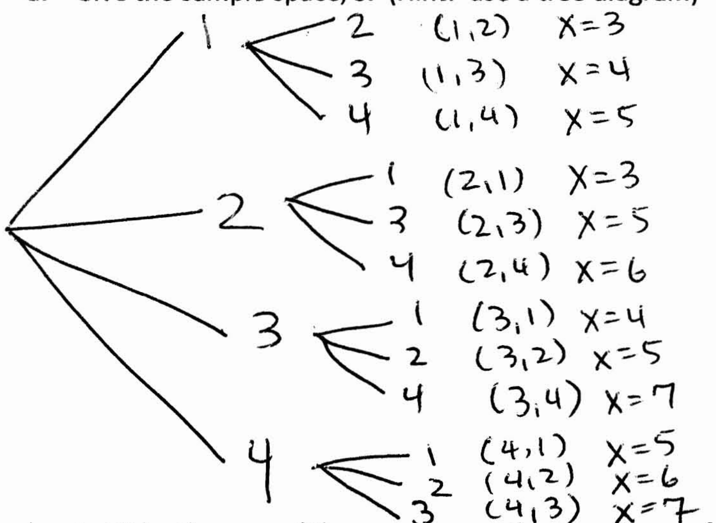
Statistics 1, Section 6 Name: Solutions

Exam 2 **November 3, 2008**

Calculators and one 8.5" by 11" sheet of handwritten notes allowed. Show all work and answers clearly in the space provided. Partial credit can only be determined based on the work shown on this exam paper. Tables are included at the end of the exam.

1. (20 points) The numbers 1,2,3 and 4 are written on four pieces of paper. Suppose two of the pieces of paper are randomly selected from a hat ***without*** replacement.

   a. Give the sample space, S. (Hint: use a tree diagram)



12 outcomes equally likely

   b. Let X be the sum of the numbers on the two pieces of paper selected. Give the probability distribution of X.

x	p(x)
3	$2/12 = 1/6$
4	$2/12 = 1/6$
5	$4/12 = 2/6$
6	$2/12 = 1/6$
7	$2/12 = 1/6$

   c. Calculate the mean and standard deviation of the probability distribution in part (b).

$$\mu = 3(1/6) + 4(1/6) + 5(2/6) + 6(1/6) + 7(1/6)$$

$$= \frac{3+4+10+6+7}{6} = \frac{30}{6} = 5$$

$$\sigma^2 = (3-5)^2(1/6) + (4-5)^2(1/6) + (5-5)^2(2/6) + (6-5)^2(1/6) + (7-5)^2(1/6)$$

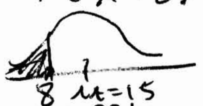
$$= \frac{4}{6} + \frac{1}{6} + 0 + 1/6 + 4/6 = \frac{10}{6}$$

$$\sigma = \sqrt{\frac{10}{6}} = 1.29$$

2. (20 points) Suppose the amount of time children watch TV per week is normally distributed with mean 15 hours and standard deviation 4 hours. If a child is randomly selected, what is the probability the child watches

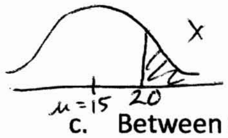
a. less than 8 hours of TV per week?

$$P(X<8) = P\left(\frac{X-\mu}{\sigma} < \frac{8-15}{4}\right) = P(Z<-1.75) = 0.0401$$



b. over 20 hours of TV per week?

$$P(X>20) = 1-P(X\le 20) = 1-P\left(Z \le \frac{20-15}{4}\right)$$
$$= 1-P(Z\le 1.25)$$
$$= 1-.8944 = .1056$$



c. Between 8 and 20 hours of TV per week

$$P(8<X<20) = P(X\le 20) - P(X\le 8) = .8944 - 0.0401$$
$$= 0.8543$$

3. (20 points) In 2001, 21% of the people in the world lived in extreme poverty (less than $1 US dollar per day). If twenty people are randomly selected from the world, what is the probability that

X = # of 20 live in extreme poverty

a. Exactly 4 live in extreme poverty

X is binom. $n=20$ $p=0.21$

$$P(X=4) = C^{20}_{4}\, p^4 q^{20-4} = (4845)(0.21^4)(0.79^{16})$$
$$= 0.217$$

b. 2 or more live in extreme poverty

$$P(X\ge 2) = 1-P(X\le 1) = 1-P(0)-P(1)$$
$$= 1 - C^{20}_{0}\, p^0 q^{20} - C^{20}_{1}\, p^1 q^{20-1}$$
$$= 1 - 1\cdot 1\cdot(.79)^{20} - 20(0.21)(0.79)^{19}$$
$$= 1 - 0.008965 - 0.0477$$
$$= 0.943$$

4. (20 points) In a certain population, 40% of people received a flu shot while 60% did not receive the flu shot. Of the people who got the flu shot, 35% caught the flu. Of the people who did not get the flu shot, 55% got the flu.

S = person got the shot F = person got the flu

a. What percent of people in the entire population got the flu?

	F	F^c	Total
S	140	260	400
S^c	330	270	600
Total	470		1000

$$P(F) = \frac{470}{1000} = .47$$

-or- $P(F) = 0.40(.35) + 0.60(.55)$

$= .14 + .33 = 0.47$

b. Given that a person caught the flu, what is the probability he/she did not get the shot?

$$P(S^c|F) = \frac{P(S^c \cap F)}{P(F)} = \frac{330}{470} = \frac{33}{47}$$

-OR- $$P(S^c|F) = \frac{P(S^c) \cdot P(F|S^c)}{P(F)}$$

$$= \frac{0.6(.55)}{.47} = .702$$

5. (10 points) A balanced die is tossed 600 times. X = the number of sixes in the 600 tosses.

   a. What are the mean and standard deviation of X? X binomial $n=600$ $p=1/6$

   $\mu = np = 600(1/6) = 100$

   $\sigma = \sqrt{npq} = \sqrt{600(1/6)(5/6)} = \sqrt{83.3} = 9.1$

   b. Would it be unusual to observe 150 sixes?

   $\mu \pm 3\sigma$

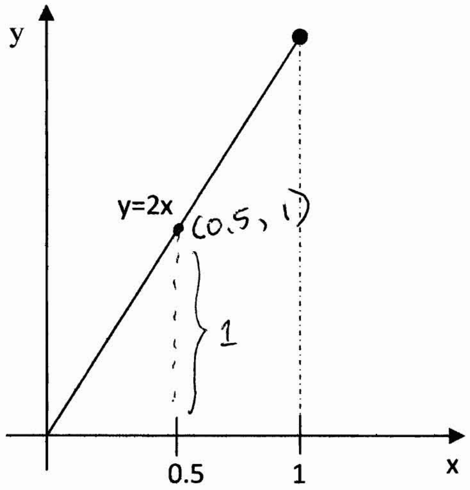
   $100 \pm 3(9.1)$

   100 ± 27.3 $(72.7, 127.3)$ since 150 is outside this interval it is very unusual

   c. How many outcomes of this experiment have 2 sixes and 598 non-sixes?

   $C^{600}_{2} = 179{,}700$ A LOT!

6. (10 points) Recall that probabilities for *continuous* random variables are defined in terms of areas under the probability density function (pdf). Suppose a random variable X has the pdf y=2x for $0 \leq x \leq 1$ (as shown below).



   a. What is the area under ANY pdf? 1

   b. Verify that this pdf has the appropriate area from part(a).

   Area of triangle $= \frac{1}{2}bh = \frac{1}{2}(1)(2) = 1$ height $=$ ~~B~~ $y = 2(1) = 2$

   c. Calculate $P(X < 0.5)$ = area of Δ with base the interval between the origin & 0.5 $= \frac{1}{2}(0.5)(1) = \frac{1}{4}$

   d. What is $P(X=0.5)$? 0