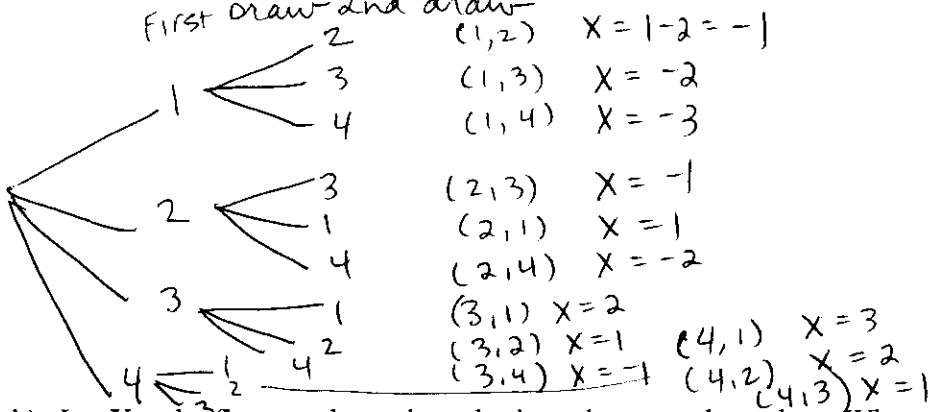
**Statistics 1, Section 2** **Name** Solutions
**Exam 3** **December 4, 2009**

*Calculators and one 8.5" by 11" sheet of handwritten notes allowed. Show all work and answers clearly in the space provided. There are 100 points possible.*

1) (20 points) The numbers 1,2,3 and 4 are written on four pieces of paper. Suppose two of the pieces of paper are randomly selected from a hat **WITHOUT** replacement.

a) List the 12 outcomes in the sample space, S. (Hint: use a tree diagram)




b) Let X = the first number selected minus the second number. What are the possible values of X? (Hint: Consider the value of X for each possible outcome in S. X can be negative.)

$X \in \{-3, -2, -1, 1, 2, 3\}$

c) Give the probability distribution of X. Note: all 12 outcomes in S are equally likely.

| x | p(x) |
|---|---|
| -3 | 1/12 |
| -2 | 2/12 |
| -1 | 3/12 |
| 1 | 3/12 |
| 2 | 2/12 |
| 3 | 1/12 |

2) (20 pts.) Suppose the heights of adult giraffes are normally distributed with a mean of 15.5 feet and standard deviation of 0.8 feet.

If a giraffe is randomly selected, what is the probability its height is

a) Less than 16 feet

$$P(X<16) = P\left(Z < \frac{16-15.5}{0.8}\right) = P(Z<0.625)$$
$$= P(Z<0.63)$$
$$= 0.7357$$

b) Between 14 and 16 feet

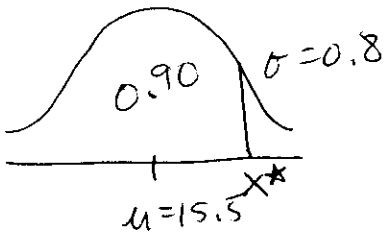
$$P(14<X<16) = P\left(\frac{14-15.5}{0.8} < Z < \frac{16-15.5}{0.8}\right)$$
$$= P(-1.875 < Z < 0.625)$$
$$= P(Z<0.625) - P(Z<-1.875)$$
$$= 0.7357 - 0.0301 = 0.7056$$

c) Over 17 feet 6 inches.

17 ft. 6 in. = 17.5 ft.

$$P(X>17.5) = P\left(Z > \frac{17.5-15.5}{0.8}\right)$$
$$= P(Z>2.50)$$
$$= 1 - P(Z \le 2.50) = 1 - 0.9938 = 0.0062$$

d) Find the 90th percentile of giraffe heights.




$$\frac{x^* - \mu}{\sigma} = z^*$$
$$\frac{x^* - 15.5}{0.8} = 1.28$$
$$x^* = 15.5 + 0.8(1.28)$$
$$x^* = 16.524 \text{ ft.}$$

3) (20 points) Suppose an author submits a 200-page book manuscript to a publisher. Let X = the number of typographical errors (typos) on a page. The probability distribution of X is shown below.

| Number of typos on page | Probability p(x) | $x \cdot p(x)$ | $x - \mu$ | $(x-\mu)^2 \cdot p(x)$ |
|---|---|---|---|---|
| 0 | 0.90 | 0 | $-0.16$ | $(-0.16)^2(0.9) = 0.02304$ |
| 1 | 0.05 | 0.05 | 0.84 | 0.03528 |
| 2 | ?0.04 | 0.08 | $2-0.16 = 1.84$ | 0.1354 |
| 3 | 0.01 | 0.03 | $3-0.16 = 2.84$ | 0.0807 |
| | | $0.16 = \mu$ | | $0.274376 = \sigma^2$ |

3 a) What is the probability of 2 typos on a randomly selected page? (You may assume no page contains more than 3 typos.)

$1 - (0.90 + 0.05 + 0.01) = 1 - 0.96 = 0.04$

7 $\mu$
7 $\sigma$

b) Calculate the mean and standard deviation of X.

$\mu = \sum x \cdot p(x) = 0.16$

$\sigma^2 = 0.274376$

$\sigma = \sqrt{0.274376} = 0.5238$

3 c) Estimate the number of typos in the entire 200-page manuscript.

$200 \cdot \mu$ since $\mu$ = average # of typos on a page

$200(0.16) = 32$ typos

4) (20 points) Let Z be the standard normal random variable. Compute the following:

a) $P(Z < 0.39)$ $= 0.6517$

b) $P(Z \geq -1.27)$ $= 1 - P(Z < -1.27) = 1 - 0.1020 = 0.898$

c) $P(Z = 3.25) = 0$

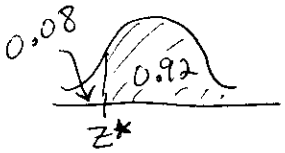
d) $P(-0.97 < Z < 3.06) = P(Z < 3.06) - P(Z < -0.97)$

$= 0.9989 - 0.1660 = 0.8329$

e) Find the value, $z^*$, of the standard normal random variable so that $P(Z < z^*) = 0.81$

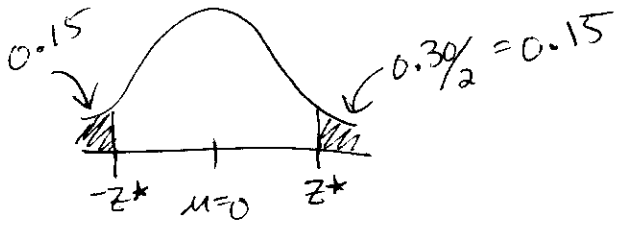
$z^* = 0.88$

f) $P(Z > z^*) = 0.92$




$z^* = -1.41$

g) $P(Z < -z^* \text{ or } Z > z^*) = 0.30$




$-z^* = -1.04$ $\quad z^* = 1.04$

5) (20 pts.) Recent research indicates that 10% of senior citizens (people over 65 years old) have Alzheimer's Disease. Suppose a group 15 senior citizens take a trip to Alaska. Assuming the group consists of randomly selected seniors, what is the probability

a) Exactly 2 seniors have Alzheimer's Disease

$X$ = # of the 15 seniors who have Alzheimer's Disease

$X$ is binomial $n=15$, $p=0.1$

$$P(X=2) = C_2^{15}\, 0.1^2 (0.9)^{15-2}$$

$$= 105 \cdot (2.54\times10^{-3}) = \boxed{0.2669}$$

b) At least 2 have Alzheimer's Disease.

$$P(X\geq 2) = 1 - P(X\leq 1)$$

$$= 1 - (P(X=0) + P(X=1))$$

$$= 1 - (0.9^{15} + 15\cdot 0.1^1 (0.9)^{14})$$

$$= 1 - (0.2059 + 0.3432)$$

$$= 0.549 = \boxed{0.4509}$$

c) Suppose in another country, say Sweden, 13% of senior citizens have Alzheimer's Disease. What is the probability exactly 3 of 15 randomly selected Swedish seniors have Alzheimer's Disease.

$$P(X=3) = C_3^{15}\, 0.13^3 (1-0.13)^{12}$$

$$= 455 \cdot (4.131\times10^{-4})$$

$$= 0.18796 \approx \boxed{0.188}$$

d) How many ways can 7 successes be arranged in 15 binomial trials? (Hint: one way is SSSSSSSFFFFFFFF, but many others exist.)

$$C_7^{15} = 6435$$