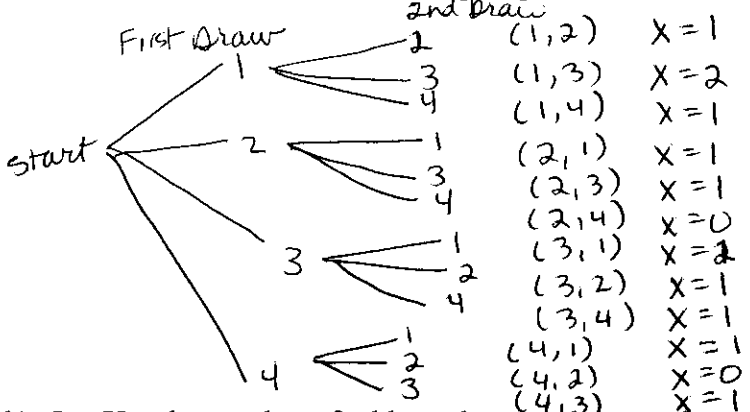
Statistics 1, Section 6 **Name** Solutions
Exam 3 **December 4, 2009**

Calculators and one 8.5" by 11" sheet of handwritten notes allowed. Show all work and answers clearly in the space provided. There are 100 points possible.

1) (20 points) The numbers 1, 2, 3 and 4 are written on four pieces of paper. Suppose two of the pieces of paper are randomly selected from a hat **WITHOUT** replacement.

a) List the 12 outcomes in the sample space, S. (Hint: use a tree diagram)



b) Let X = the number of odd numbers on the two pieces of paper selected. What are the possible values of X? (Hint: Consider the value of X for each possible outcome in S. If you drawn a 3, then a 4, then X = 1 since there was one odd number among the two drawn.)

0, 1, 2

c) Give the probability distribution of X. Note: all 12 outcomes in S are equally likely.

x	p(x)
0	2/12
1	8/12
2	2/12

2) (20 pts.) Suppose Acme Airlines, Inc. claims that their flights arrive at their destinations on time 95% of the time. Suppose that you have recently taken 11 flights with Acme and that 95% of their flights do arrive on time, what is the probability that

a) All 11 flights arrive on time ?

X = # flights out of 11 that arrive on time

$P(X=11) = 0.95^{11} = 0.5688$

b) At most 9 of the flights arrive on time?

$$P(X \leq 9) = 1 - P(X \geq 10)$$
$$= 1 - (P(X=10) + P(X=11))$$
$$= 1 - (C_{10}^{11}(0.95)^{10}(0.05)^{1} + 0.5688)$$
$$= 1 - (0.3293 + 0.5688) = 0.1019$$

c) Suppose ABC Airlines claims that 97% of their flights arrive on time. What is the probability exactly 10 of 11 ABC flights arrive on time?

$$P(X=10) = C_{10}^{11}(0.97)^{10}(0.03)^{11-10}$$
$$= 0.2433$$

d) How many ways can 8 successes be arranged in 11 binomial trials?

$$C_8^{11} = 165$$

3) (20 points) Let X = the net winnings for the player in a certain casino game. The probability distribution of X is shown below. Negative winnings mean the player has lost money

X = net winnings	Probability p(x)	$x \cdot p(x)$
-$10	0.80	-8
-$1	0.15	-0.15
$20	? 0.04	0.80
$100	0.01	$100(0.01) = 1$
		$-6.35 = \mu$

a) What is the probability the player wins $20?

$1 - (0.80 + 0.15 + 0.01) = 1 - 0.96 = 0.04$

b) Calculate the mean of X.

$\mu = -\$6.35$

c) Estimate the net winnings if the player plays 1000 rounds of this game. (Indicate losses with a negative sign.)

$1000 \cdot \mu = 1000(-6.35)$

$= -\$6350$

d) Calculate the standard deviation of the following probability distribution:

x	p(x)	$x \cdot p(x)$	$x - \mu$	$(x-\mu)^2 \cdot p(x)$
0	0.1	0	-1.2	$(-1.2)^2(0.1) = 0.144$
1	0.6	0.6	-0.2	0.024
2	0.3	0.6	0.8	0.192
		$1.2 = \mu$		$0.36 = \sigma^2$

$\sigma = \sqrt{0.36} = 0.6$

4) (20 points) Let Z be the standard normal random variable. Compute the following:

2 a) $P(Z < -2.03)$ $= 0.0212$

3 b) $P(Z \geq 0.87) = 1 - P(Z < 0.87) = 1 - 0.8078 = 0.1922$

3 c) $P(Z = 1.69) = 0$

3 d) $P(-1.01 < Z < -0.63) = P(Z < -0.63) - P(Z < -1.01)$

$= 0.2643 - 0.1562 = 0.1081$

3 e) Find the value, z^*, of the standard normal random variable so that $P(Z < z^*) = 0.38$

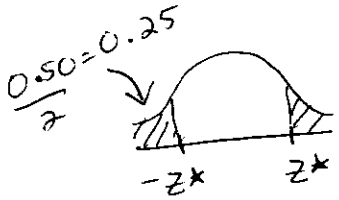
$z^* = -0.31$

3 f) $P(Z > z^*) = 0.76$

$P(Z \leq z^*) = 1 - 0.76 = 0.24$

$z^* = -0.71$

3 g) $P(Z < -z^* \text{ or } Z > z^*) = 0.50$



$= -0.67$

$z^* = 0.67$

5) (20 pts.) The amount of aspirin in an aspirin tablet is normally distributed with mean 201 milligrams and standard deviation 10 milligrams. $\sigma = 10$ $\mu = 201$

What is the probability a randomly selected aspirin tablet contains

a) Less than 190 milligrams of aspirin X = amt. of aspirin in a tablet

$$P(X<190) = P\left(Z < \frac{190-201}{10}\right) = P(Z < -1.10)$$

$$= 0.1357$$

b) Between 190 and 210 milligrams of aspirin

$$P(190 < X < 210) = P\left(-1.10 < Z < \frac{210-201}{10}\right)$$

$$= P(-1.10 < Z < 0.90)$$

$$= P(Z < 0.90) - P(Z < -1.10)$$

$$= 0.8159 - 0.1357 = 0.6802$$

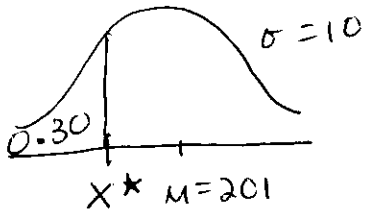
c) Over 220 milligrams of aspirin

$$P(X > 220) = P\left(X > \frac{220-201}{10}\right)$$

$$= P(X > 1.90) = 1 - P(X \le 1.90)$$

$$= 1 - 0.9713 = 0.0287$$

d) Find the 30th percentile of the amount of aspirin in these tablets.



$$z^* = -0.52$$

$$\frac{x^* - \mu}{\sigma} = z^*$$

$$\frac{x^* - 201}{10} = -0.52$$

$$x^* = 201 + 10(-0.52)$$

$$x^* = 195.8 \text{ mg}$$