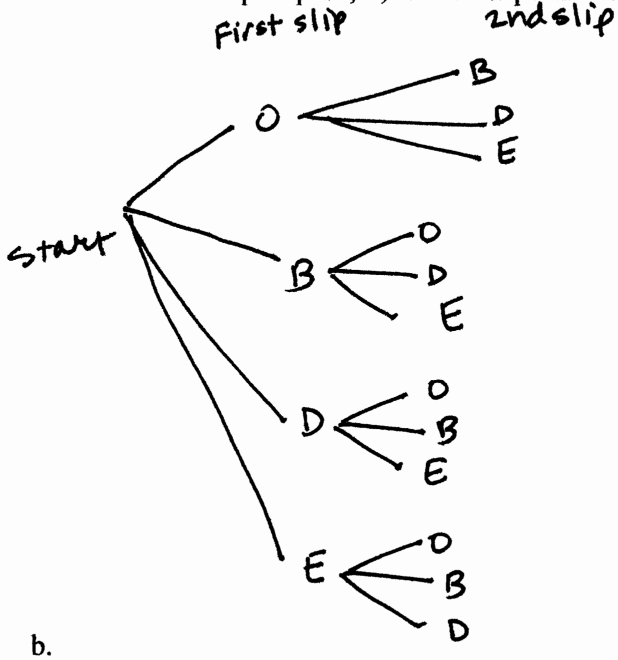
Statistics 1 (Norris) Name Solution
Exam 2 November 4, 2009

Calculators and one 8.5" by 11" sheet of handwritten notes allowed. Show all work and answers clearly in the space provided. There are 80 points possible.

1. (15 points) The letters O, B, D and E are written on four slips of paper and put into a hat. After mixing the slips, two slips are randomly selected WITHOUT replacement.

   a. Give the sample space, S, for this experiment. (Hint: A tree diagram might help.)



$S = \{OB, OD, OE, BO, BD, BE, DO, DB, DE, EO, EB, ED\}$

12 outcomes in S, equally likely

   b.

Event	List the outcomes in the event	Probability of the event
B is one of the selected letters	OB, BO, BD, BE, DB, EB	$\frac{6}{12} = \frac{1}{2}$
The two letters form a word in English	BE, DO	$\frac{2}{12} = 1/6$
The letters are drawn in alphabetical order	Not required BO, BD, BE, DO, DE, ED	$\frac{6}{12} = 1/2$
Two vowels are drawn	OE, EO	$\frac{2}{12} = 1/6$
At least one consonant is drawn	Not required	$1 - 1/6 = 5/6$

2. (15 points) A group of 250 subjects are classified by the amount they exercised per week and also by whether or not they got a cold when exposed to a virus. The results are shown below:

	Cold	No Cold	Total
Little to no exercise	55	45	100
Moderate exercise	51	49	100
Significant exercise	17	33	50
	123	127	250

Assume a person is randomly selected from this group, find the probability

a. The person gets little to no exercise. $\frac{100}{250} = \frac{10}{25} = 0.4$

b. The person caught the cold. $\frac{123}{250}$ $= .492$

c. The person gets little to no exercise and caught the cold $\frac{55}{250}$ $= 0.22$

d. The person caught the cold given they get little to no exercise

$$P(\text{cold} \mid \text{little exer}) = \frac{55}{100}$$

e. The person gets moderate exercise given they had no cold.

$$P(\text{mod ex} \mid \text{no cold}) = \frac{49}{127} = 0.386$$

f. The person caught the cold or gets little to no exercise

$$P(\text{cold}) + P(\text{little ex}) - P(\text{cold} \cap \text{little ex})$$

$$\frac{123}{250} + \frac{100}{250} - \frac{55}{250} = \frac{168}{250} = 0.672$$

$123 + 45 = 168$

g. Are the events "little to no exercise" and "cold" independent? Justify your answer (no credit unless you justify it correctly.).

$$P(\text{cold}) = \frac{123}{250}$$

$$P(\text{cold} \mid \text{little to no exer}) = \frac{55}{100}$$

not equal

Thus, cold & little to no exer. not indept

3. (15 points) A four-member interviewing committee is being selected from a pool of 30 employees at a company. Of the employees, 20 are female and 10 are male. If the committee is randomly selected, what is the probability

4 a. All four members are female $\frac{20}{30} \cdot \frac{19}{29} \cdot \frac{18}{28} \cdot \frac{17}{27} = 0.177$

4 b. All four members are the same gender $P(MMMM) + P(FFFF)$

$\frac{10}{30} \cdot \frac{9}{29} \cdot \frac{8}{28} \cdot \frac{7}{27} +$

$0.00766 + 0.177 = 0.185$

4 c. P(At least one member is male) $= 1 - P(\text{no males})$

$= 1 - P(FFFF)$

$= 1 - 0.177 = 0.823$

4 Extra credit d. Exactly one male

$P(MFFF) + P(FMFF) + P(FFMF) + P(FFFM)$

$= 4 \cdot P(MFFF)$

$= 4\left(\frac{10}{30}\right)\left(\frac{20}{29}\right)\left(\frac{19}{28}\right)\left(\frac{18}{27}\right) = 0.416$

3 e. Committee is all female, given the committee is all the same gender

$P(\text{all female} \mid \text{all same gender}) = \frac{P(\text{all female} \cap \text{all same gender})}{P(\text{all same gender})}$

$= \frac{P(\text{all female})}{P(\text{all same})} = \frac{0.177}{0.185} = 0.957$

4. (10 points) Consider an experiment having sample space, $S = \{e_1,e_2,e_3,e_4,e_5\}$. Suppose the probability of e_1 is 0.1 and that $P(e_1)=P(e_3)=P(e_5)$. Also, the probability of e_2 is three times the probability of e_4. Find

✓

a. $P(e_4) = 0.175$

b. $P(e_2) = 0.525$

c. $P(\{e_1,e_2,e_4\})$

$= 0.1 + 0.525 + 0.175$

$= 0.8$

$P(e_2) = 3 \cdot P(e_4)$

$P(e_1) + P(e_2) + P(e_3) + P(e_4) + P(e_5) = 1$

$(0.1) + 3P(e_4) + 0.1 + P(e_4) + 0.1 = 1$

$4P(e_4) + 0.3 = 1$

$4P(e_4) = 0.7$

$P(e_4) = \frac{0.7}{4} = 0.175$

$P(e_2) = 3(.175) = 0.525$

5. (10 points) Suppose P(B)=0.3, P(A ∩B)=0.05, P(AUB)=0.7.
Calculate

a. P(A)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.7 = P(A) + 0.3 - 0.05$

$0.7 = P(A) + 0.25 \Rightarrow P(A) = 0.45$

b. $P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.05}{0.45} = \dfrac{5}{45} = \dfrac{1}{9}$

c. Are A and B independent? Give reasons for credit.

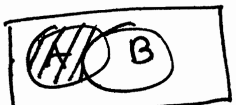
$P(B) = 0.3 \neq P(B \mid A) = 1/9 \approx 0.11$

A & B are <u>NOT</u> indept.

d. Are A and B mutually exclusive? Give reasons for credit.

No. $P(A \cap B) = 0.05$ So A & B ~~have~~ can happen at the same time

e. P(A ∩ (B^C)) (Hint: A Venn diagram might help.)



$A \cap B^c$ is shaded $P(A \cap B^c) = P(A) - P(A \cap B) = 0.45 - 0.05 = 0.40$

6. (15 points) A person plays a game of roulette. The probability of winning on any single round is 18/38. Suppose the person plays two rounds of roulette. Calculate the probability

5 a. She wins on both rounds.

$\left(\frac{18}{38}\right)\left(\frac{18}{38}\right) = 0.224$

5 b. She wins exactly one round.

$\left(\frac{18}{38}\right)\left(\frac{20}{38}\right) + \left(\frac{20}{38}\right)\left(\frac{18}{38}\right) = 0.4986$

$P(W L) + P(LW)$

5 c. If she plays 10 rounds, what is the probability she LOSES on all 10 rounds?

$\left(\frac{20}{38}\right)^{10} = 0.001631$

5 d. Extra Credit: If she plays 10 rounds, what is the probability she wins <u>exactly</u> 9 rounds?

$10 \times \left[\left(\frac{18}{38}\right)^9\left(\frac{20}{38}\right)\right] = 0.006319$