Solutions to Practice Questions for Exam 3

1. a) $S = \{ \underset{X=3}{(1,2)}, \underset{X=4}{(1,3)}, \underset{X=5}{(1,4)}, \underset{X=3}{(2,1)}, \underset{X=5}{(2,3)}, \underset{X=6}{(2,4)}, \underset{X=4}{(3,1)}, \underset{X=5}{(3,2)}, \underset{X=7}{(3,4)}, \underset{X=5}{(4,1)}, \underset{X=6}{(4,2)}, \underset{X=7}{(4,3)} \}$

b)

x	$p(x)$
3	$2/12 = 1/6$
4	$2/12 = 1/6$
5	$4/12 = 2/6$
6	$2/12 = 1/6$
7	$2/12 = 1/6$
Total	$12/12$

There are 12 equally likely outcomes

c)

x	$p(x)$	$x \cdot p(x)$	$(x-\mu)$	$(x-\mu)^2 \cdot p(x)$
3	$1/6$	$3/6$	$3-5=-2$	$(-2)^2(1/6) = 4/6$
4	$1/6$	$4/6$	$4-5=-1$	$(-1)^2(1/6) = 1/6$
5	$2/6$	$10/6$	$5-5=0$	$0^2(2/6) = 0$
6	$1/6$	$6/6$	$6-5=1$	$1^2(1/6) = 1/6$
7	$1/6$	$7/6$	$7-5=2$	$2^2(1/6) = 4/6$
		$\mu = \Sigma x \cdot p(x) = 30/6 = 5$		$10/6 = \sigma^2$

$\mu = \Sigma x \cdot p(x) = 30/6 = 5$

$\sigma^2 = \Sigma (x-\mu)^2 \cdot p(x) = 10/6 = 5/3 = 1\,2/3$

$\sigma = \sqrt{5/3} = 1.29$

2. a) $P(Z < 0.79) = 0.7852$

b) $P(Z > -1.94) = 1 - 0.0262 = 0.9738$

c) $P(-1.20 < Z < 2.08) = P(Z < 2.08) - P(Z < -1.20) = 0.9812 - 0.1151 = 0.8661$

d) $P(Z = 1.05) = 0$

e) $P(Z < z^*) = 0.35$ ~~≈ 0.3409~~ $z^* = -0.39$

2 f) $P(Z < z^*) = 0.70 \qquad z^* = 0.52$

g) $P(Z > z^*) = 0.85$

$P(Z \leq z^*) = 1 - .85 = 0.15$

$z^* = -1.04$

3. a) 5 ft. 11 inches $= 5(12) + 11 = 60 + 11 = 71$ inches

X = height of a randomey selected woman

$$P(X > 71) = P\left(Z > \frac{71-\mu}{\sigma}\right)$$

$$= P\left(Z > \frac{71-64}{2.6}\right)$$

$$= P(Z > 2.69)$$

$$= 1 - P(Z \leq 2.69)$$

$$= 1 - 0.9964 = 0.0036$$

b) $$P(62 < X < 67) = P\left(\frac{62-\mu}{\sigma} < Z < \frac{67-\mu}{\sigma}\right)$$

$$= P\left(\frac{62-64}{2.6} < Z < \frac{67-64}{2.6}\right)$$

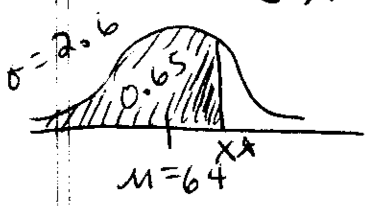
$$= P(-0.77 < Z < 1.15)$$

$$= P(Z < 1.15) - P(Z < -0.77)$$

$$= 0.8749 - 0.2206 = 0.6543$$

c) x^* = 65th percentile

$P(X \leq x^*) = 0.65$



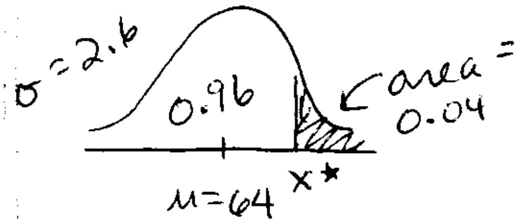
$$\frac{x^* - \mu}{\sigma} = z^*$$ ← z-value cuts of 0.65 area below

$$\frac{x^* - 64}{2.6} = 0.39$$

$$x^* = 64 + 2.6(0.39)$$

$x^* = 65.01 \approx 65$ inches.

3. d)



$$\frac{x^* - \mu}{\sigma} = z^*$$ ← cuts off area 0.96 below

$$\frac{x^* - 64}{2.6} = 1.75$$

$$x^* = 64 + 2.6(1.75)$$

$x^* = 68.55$ inches

= 5 feet 8.55 inches

e) Empirical Rule is applicable since data are normal

$\mu \pm 2\sigma$ contains 95% of probability

$64 \pm 2(2.6)$

64 ± 5.2

$64 - 5.2, \; 64 + 5.2$

(58.8 in., 69.2 inches)

95% of women will have height between 58.8 and 69.2 inches.

4. Binomial Experiment ~~n=20~~

X = # who live in extreme poverty out of the 20 randomly selected

$n = 20, \quad p = 0.21$

a) $P(X=4) = C_4^{20}(0.21)^4(1-0.21)^{16} = \cancel{0.586}\ 0.217$

b) $P(X \geq 2) = 1 - P(X \leq 1)$

$= 1 - [P(X=0) + P(X=1)]$

$= 1 - [0.79^{20} + C_1^{20}(0.21)^1(0.79)^{19}]$

$= 1 - [0.009 + 0.048] = 0.943$

4 c) $P(2 \le X \le 4) = P(X=2) + P(X=3) + P(X=4)$

$= C_2^{20}\, 0.21^2 (0.79)^{18} + C_3^{20}\, 0.21^3 (0.79)^{17} + C_4^{20} (0.21)^4 (0.79)^{16}$

$= 0.120 + 0.192 + 0.217$

$= 0.529$

5. X is a binomial RV

$n = 600$ p = prob of a "six" $= 1/6$

a) $\mu = np = 600(1/6) = 100$ "sixes"

$\sigma^2 = np(1-p) = 600(1/6)(1-1/6)$

$= 600(1/6)(5/6)$

$= 100(5/6) = 500/6$

$\sigma = \sqrt{500/6} = 9.13$

b) conside $\mu \pm 2\sigma$ since about 95% of the time X falls in this interval

$\mu \pm 2\sigma = 100 \pm 2(9.13)$

100 ± 18.26

$(81.74,\ 118.26)$ "sixes"

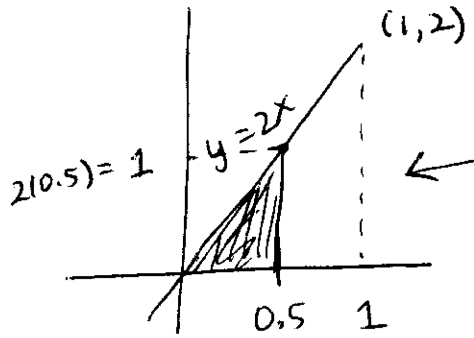
95% of the time, 600 tosses will yield between 82 and 118 "sixes" so 150 "sixes" is unusual.

c) $C_2^{598} = 178{,}503$ outcomes have 2 "sixes" and 598 non-sixes.

6. a) 1 b) area under pdf $= \frac{1}{2}(\text{base}) \times (\text{height})$

$= \frac{1}{2}(1)(2) = 1$

6c) $P(X<0.5)$ = area under pdf $y=2x$ to the left of 0.5 (shown here)



$= \frac{1}{2}(\text{base})\times(\text{height})$

$= \frac{1}{2}(0.5)(1)$

$= \frac{1}{2}(\frac{1}{2})(1) = 1/4$

d) $P(X=0.5)=0$

7. a) $P(\text{win}) = P(\text{sum is 3 or less}) = P(\{(1,1),(1,2),(2,1)\})$

X	$p(x)$
-\$5	$1-3/36 = 33/36 = 11/12$
\$25	$3/36 = \frac{1}{12}$

There are 36 outcomes, all equally likely when a pair of fair dice are tosses

so $P(\text{win}) = 3/36$

b) $\mu = \sum x \cdot p(x) = (-5)(11/12) + 25(\frac{1}{12})$

$= -55/12 + 25/12 = \frac{-30}{12} = -\frac{5}{2} = -2.50$

on average, you lose \$2.50 per play.

c) $1000(-2.50) = -\$2500$

d)

	X	$p(x)$	$x \cdot p(x)$
lose	$-r$	$11/12$	$-r(11/12)$
win	$30-r$	$1/12$	$(30-r)(\frac{1}{12})$

r = amount casino should charge so $\mu = \$0$

$-\frac{11}{12}r + (30-r)(\frac{1}{12}) = 0$

$-\frac{11}{12}r + \frac{30}{12} - \frac{1}{12}r = 0$

$-\frac{12}{12}r + \frac{30}{12} = 0$

$-r + 30/12 = 0$

$r = 30/12 = \$2.50$