A Dynamic Model of the Foreign Exchange Market

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Abstract

This paper offers a dynamic model of the foreign exchange market with asymmetric information. The main result is that level of asymmetric information in the market offers a lot of insight with respect to the size of disconnect between the exchange rate and macroeconomic fundamentals.

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1 Introduction

It has been well-established in the exchange rate literature that macroeconomic variables have very little effect on floating exchange rates, especially in the short-run. There seems to be a disconnect between macroeconomic fundamentals and the exchange rate. A notable demonstration of this disconnect is by Flood and Rose (1995). Their conclusion is that most critical determinants of exchange rate volatility are not macroeconomic and instead, research should concentrate on more microeconomic detail.

This study builds on the model introduced in Onur (2008), which is a myopic model of the foreign exchange market with the required microeconomic detail (microstructure). In this paper, I extend that myopic model to a dynamic case, which allows for the analysis of the disconnect between macroeconomic fundamentals and the exchange rate[1]. This disconnect has been demonstrated by Bacchetta and van Wincoop (2006) to stem from information dispersion. Their explanation is that when there are heterogeneously informed investors in a standard exchange rate model, this causes rational confusion such that investors are confused about whether the change in exchange rate is caused by a change in fundamentals or by a change in non-fundamentals. What this study shows is that the magnitude of this disconnect is very much dependent on the amount of asymmetric information in the market. The most striking result is that it takes very little amount of private information to be present in the market to cause considerable amount of disconnect[2].

The rest of the paper is arranged as follows. Section 2 sets up and solves the model. Section 3 analyzes the results and concludes.

2 Theoretical Model

The setup of the model follows from Onur (2008). It is a two-country monetary model of exchange rate determination with money market equilibrium, purchasing power parity and interest rate

[1] Bacchetta and van Wincoop (2006) refer to the same idea as "the exchange rate determination puzzle."

[2] Wang (1993) also finds a similar result for equity markets.

parity assumed.

There is a continuum of investors in both countries and they are distributed on the $[0,1]$ interval. I assume a myopic agent setup where agents live for two periods and make only one investment decision. Investors are identical in the sense that they have the same utility function and they know that exchange rate depends on the expectations of future fundamentals.

Investors in both countries can invest in money of their own country, bonds of the home country for a return of i_t, bonds of the foreign country for a return of i_t^*, and in some type of production with a fixed return. This production is assumed to depend on the exchange rate as well as on real money holdings of investor i, μ_t^i. Thus, the production function is written as $f\left(\mu_t^i\right) = \kappa_t^i s_{t+1} - \mu_t^i \left(\ln\left(\mu_t^i\right) - 1\right)/\alpha$, for $\alpha > 0$. The coefficient κ_t^i is the exchange rate exposure variable. Investor i will want to hedge himself, and this hedge against non-asset income will add to the demand in the foreign exchange market. An investor's hedge demand changes every period and it is known by the investor himself only.

Investor i maximizes his expected discounted future utility conditional on information known at t, F_t^i, and his budget constraint. The maximization problem can be written as

$$\begin{aligned} \max \quad & -E_t\left[e^{-\gamma c_{t+1}^i} | F_t^i\right] \qquad (1) \\ \text{subject to } c_{t+1}^i \quad = \quad & (1+i_t)w_t^i + (s_{t+1} - s_t + i_t^* - i_t)B_t^i - i_t\mu_t^i + f\left(\mu_t^i\right), \end{aligned}$$

where w_t^i is wealth at the start of period t, B_t^i is the amount invested in foreign bonds, and $s_{t+1} - s_t + i_t^* - i_t$ is the log-linearized excess return on investing in foreign bonds. Investor i chooses the optimal amount of foreign bonds to hold and the first order condition becomes

$$s_t = E^i(s_{t+1}) - i_t + i_t^* - \gamma\sigma_{t,i}^2\left(B_t^i + b_t^i\right), \qquad (2)$$

where $\sigma_{t,i}^2 = var(s_{t+1})$ is the conditional variance of next period's exchange rate and b_t^i is the hedge demand due to the exchange rate exposure of non-asset income, $b_t^i = \kappa_t^i$. I write the interest differential in terms of the exchange rate and fundamentals to obtain $i_t - i_t^* = \frac{1}{\alpha}(s_t - f_t)$, where the fundamentals are defined as $f_t = (m_t - m_t^*)$[3].

[3]Note that m_t and i_t are logs of money supply and interest rate respectively.

Investor i's foreign bond demand can be written as

$$B_t^i = \frac{E^i(s_{t+1}) - s_t + i_t^* - i_t}{\gamma \sigma_{t,i}^2} - b_t^i, \tag{3}$$

Hedging demand emerging from the exchange rate exposure variable is assumed to be composed of an average term and an idiosyncratic term, $b_t^i = b_t + \varepsilon_t^i$. The average hedging demand is unobservable to any of the investors but they know the autoregressive process it follows, $b_t = \rho_b b_{t-1} + \varepsilon_t^b$, where $\varepsilon_t^b \sim N(0, \sigma_b^2)$.

Applying this market equilibrium condition to (3) yields

$$\overline{E}_t(s_{t+1}) - s_t = i_t - i_t^* + \gamma b_t \sigma_t^2, \tag{4}$$

where $\overline{E}_t$ is the average expectation across all investors. Using equation (4) and the definition of fundamentals, the equilibrium exchange rate is given by

$$s_t = \sum_{k=0}^{\infty} \left(\frac{\alpha}{1+\alpha} \right)^k \overline{E}_t^k \left(f_{t+k} - \alpha \gamma \sigma_{t+k}^2 b_{t+k} \right), \tag{5}$$

where $\overline{E}_t^k$ are expectations of order $k > 1$, defined as $\overline{E}_t^k(s_{t+k}) = \int_0^1 E_t^i \left(\overline{E}_t^{k-1} (s_{t+k-1}) \right) di$ with $\overline{E}_t^1(s_{t+1}) = \overline{E}_t(s_{t+1})$ and $\overline{E}_t^0(s_t) = s_t$.

2.1 Information Structure

As well as observing all past and current values of fundamentals, investors also observe signals regarding future fundamentals. All investors in the market receive a noisy public signal about the future value of fundamentals. Asymmetric information arises from the fact that a proportion of investors also receive a noisy private signal about the future value of fundamentals in addition to the public signal received by every investor. I use ω to denote the proportion of investors who receive just the public signal. Throughout the paper, I choose to call them "uninformed" investors for tractability reasons. The remaining proportion of investors, $1-\omega$, are classified as "informed" investors. Note that changing the ratio of informed investors to uninformed investors in the economy is equivalent to choosing how much private information exists in the market[4].

I Assume that the fundamentals in the economy are governed by the process

$$f_t = D(L)\varepsilon_t^f, \qquad \varepsilon_t^f \sim N(0, \sigma_f^2), \tag{6}$$

where $D(L) = d_1 + d_2 L + d_3 L^2 + ...$

I also assume the noisy public signal received by all investors to be denoted by z_t and the noisy private signal received by informed investor i to be denoted by ν_t^i. These signals carry information about the value of the fundamental T periods ahead, f_{t+T}. In the economy, let the noisy public signal be structured in the following manner:

$$\begin{aligned} z_t &= f_{t+T} + w_t^z \qquad where \\ w_t^z &= \rho_z w_{t-1}^z + \varepsilon_t^z \qquad \varepsilon_t^z \sim N(0, \sigma_z^2) \end{aligned} \tag{7}$$

The public signal is composed of two components, the actual value of future fundamentals and a persistent term, w_t^z. I assume that this persistent term follows an autoregressive process with $\rho_z < 1$.[5] The error term, ε_t^z, is independent from the value of future fundamentals and it is unknown to investors at time t. In this setup, public signal does not reveal the exact value of future fundamentals to the investors. On the other hand, the structure of the noisy private signal is:

$$\nu_t^i = f_{t+T} + \varepsilon_t^{\nu i} \qquad \varepsilon_t^{\nu i} \sim N(0, \sigma_\nu^2), \tag{8}$$

where error term of the signal being independent from u_t and other investors' signals. Due to the law of large numbers, I assume the average signal received by informed investors to be u_t, namely $\int_{1-\omega}^{1} \nu_t^i di = u_t$.

[4] When $\omega = 1$, every investor in the economy is receiving *only* the public signal so all are uninformed, and when $\omega = 0$, every investors is receiving *both* of the signals so all of them are informed.

[5] The fact that z_t has a persistent term permits the public signal to be written in terms of its current and past innovations when conjecturing the equilibrium exchange rate.

2.2 The Equilibrium Exchange Rate

I adopt a solution method suggested by Townsend (1983) and successfully applied to an exchange rate model by Bacchetta and Van Wincoop (2006). The solution method realizes that the equilibrium exchange rate and its components can be represented by a combination of current and past shocks to the economy. Realizing that $f_t = D(L)\varepsilon_t^f$ and $b_t = G(L)\varepsilon_t^b$; where $D(L) = d_1 + d_2 L + d_3 L^2 ...$ and $G(L) = 1 + \rho_b L + \rho_b^2 L^2 + ...$ where L is the lag operator, equation (5) can be represented in terms of current and past innovations. The next step is to conjecture a representation for the equilibrium exchange rate in terms of current and past innovations and then use the method of undetermined coefficients solution technique to solve for the equilibrium values of the coefficients. The conjectured exchange rate depends on shocks to observable and unobservable fundamentals:

$$s_t = A(L)\varepsilon_{t+T}^f + B(L)\varepsilon_t^b + C(L)\varepsilon_t^z \tag{9}$$

where $A(L)$, $B(L)$, and $C(L)$ are infinite order polynomials in L. Only a portion of these innovations are unknown to the investor. At time t, investors observe today's fundamentals as well as innovations from previous periods. That means values of ε^f between $t+1$ and $t+T$ are unknown to the investor. Investors also do not observe the non-fundamental shocks, ε^b, between t and $t-T$, as well as the public information shocks, ε^z, between t and $t-T$. Non-fundamental and public shocks before time $t-T$ are known by the investors.

Detailed solution of method of undetermined coefficients involves using equation (9) and the distinct information structure of two types of investors in the market to compute $\overline{E}_t(s_{t+1})$ and σ_t^2. Once they are written in terms of innovations, I match their coefficients with the initial conjecture to find the equilibrium exchange rate. Note that both of the terms $\overline{E}_t(s_{t+1})$ and σ_t^2 depend on the amount of asymmetric information in the market, ω[6].

[6] A detailed description of this solution is available from the author upon request.

3 Analysis and Results

Figures 1 through 4 show the dynamic impact of one-standard-deviation shock on the exchange rate for different levels of asymmetric information in the market. The shocks are innovations to future fundamentals, innovations to aggregate hedge demand and innovations to public signal. To be able to derive direct comparison with Bacchetta and Van Wincoop (2006), I follow their lead and set standard deviations of all shocks to 0.01 except for that of public shock which is 0.08. Other parameters used are $T = 8, \alpha = 10, \gamma = 500$ and $\rho_b = 0.8$[7].

When each shock is analyzed separately, it is clear that the instantaneous response of the exchange rate to public shocks is highest in figure 4, when 90 percent of investors are receiving only public signal and only the remaining 10 percent are receiving both the public and the private signal. Naturally, magnitude of this response is lowest for figure 1 when $\omega = 0.1$. This magnitude increases as ω increases.

When the instantaneous response of the exchange rate to future fundamental shocks is analyzed, figure 1 depicts the case with the biggest response. This is true because there is more information about future fundamentals in the market due to very high proportion of the investors receiving both public and private signals. The response decreases as ω increases.

A more surprising result is the instantaneous response of the exchange rate to the change in non-fundamentals (b-shocks). It is obvious in figures 1 through 4 that impact of b-shocks is substantial in all four cases. The impact increases as ω increases since less information about future fundamentals causes a bigger (rational) confusion. When compared to the findings of Bacchetta and Van Wincoop (2006), this postulates that every investor in the market does not need to receive a private signal to create high magnitudes of disconnect. Actually, a small amount of private information in the market is enough to cause the biggest instantaneous response of exchange rates to non-fundamental shocks.

As a result, if one wanted to strengthen the connection between exchange rates and macroeconomic fundamentals, and even decrease the volatility in the market subsequently, increasing

[7] Bacchetta and Van Wincoop (2006) analyze the cases where ω is equal to 0 and 1. This model allows for analysis when ω is between 0 and 1.

the proportion of investors holding more information (maybe through information disclosure) might be a good idea.

[Figure 1]

[Figure 2]

[Figure 3]

[Figure 4]

References

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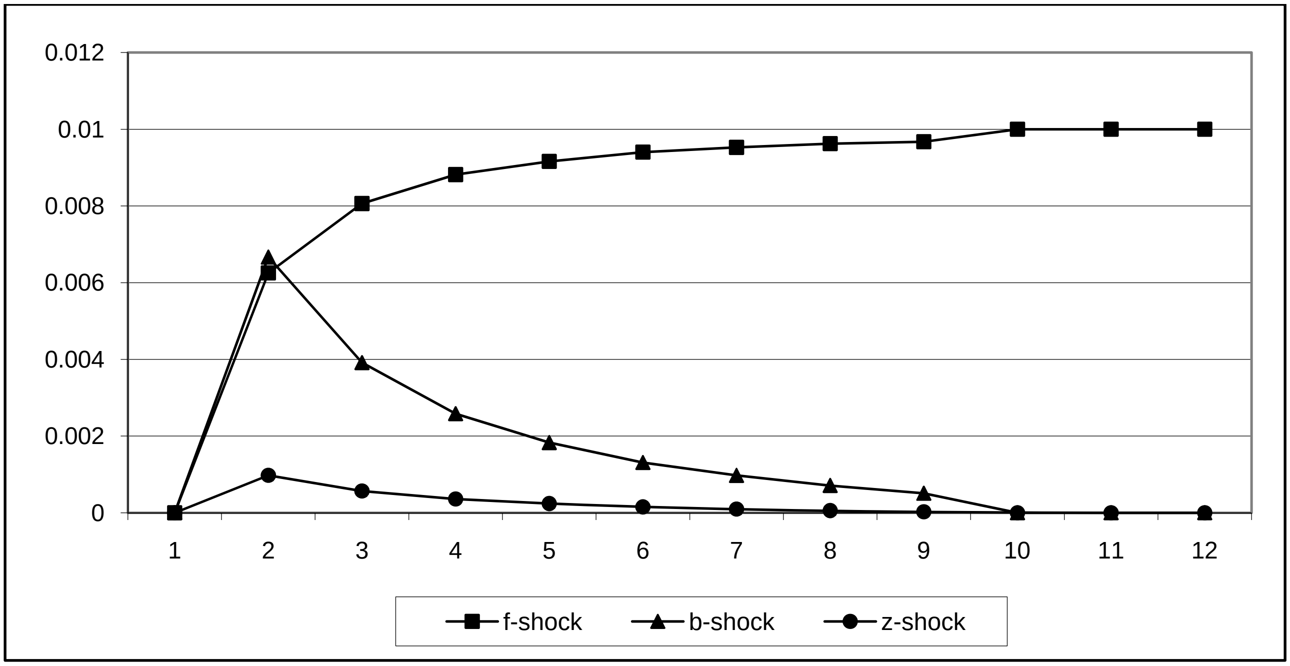

Figure 1: Impulse Response Function (ω=0.1)

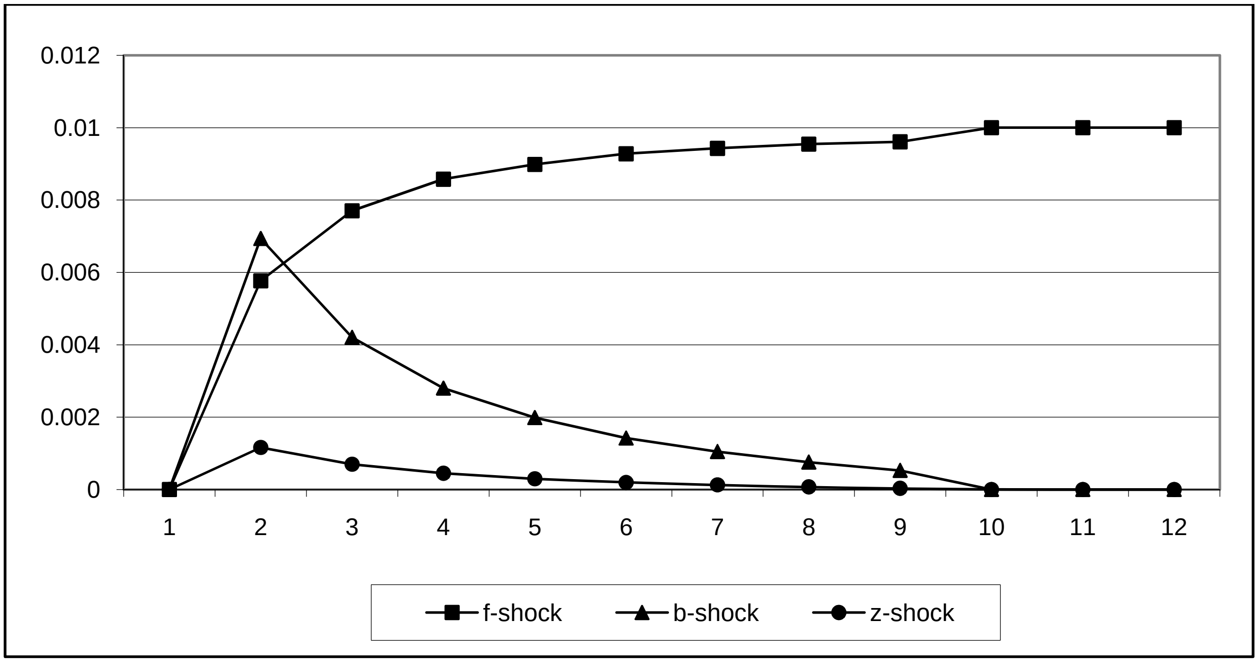

Figure 2: Impulse Response Function (ω=0.3)

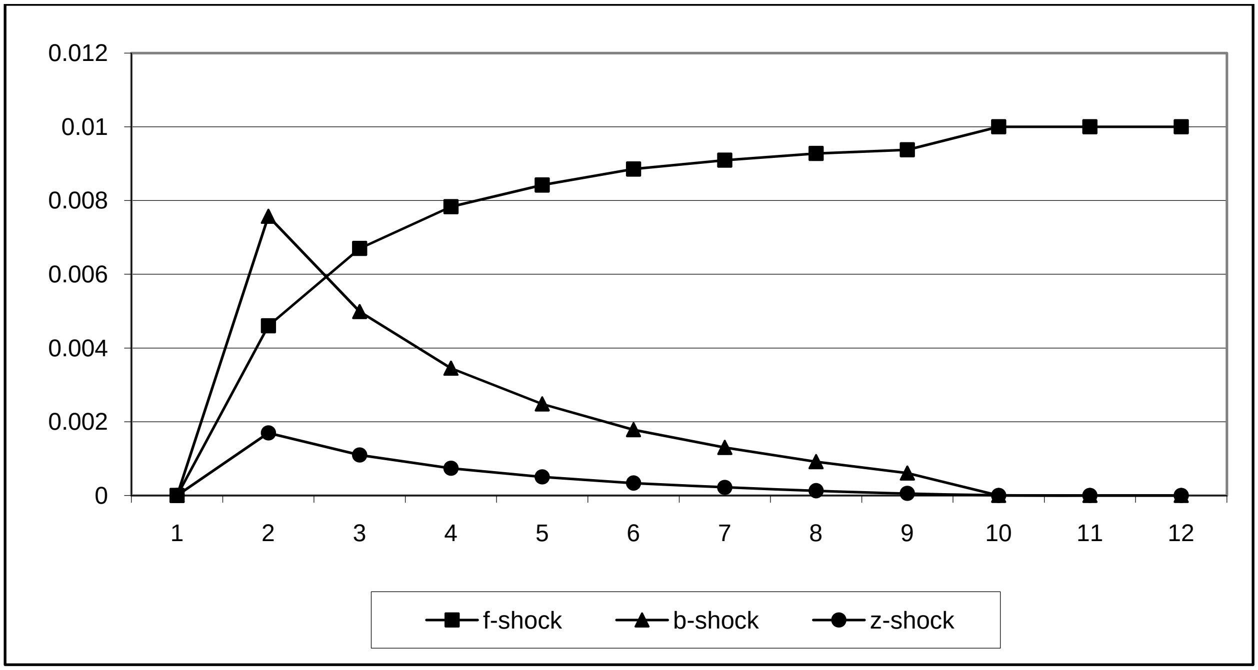

Figure 3: Impulse Response Function (ω=0.6)

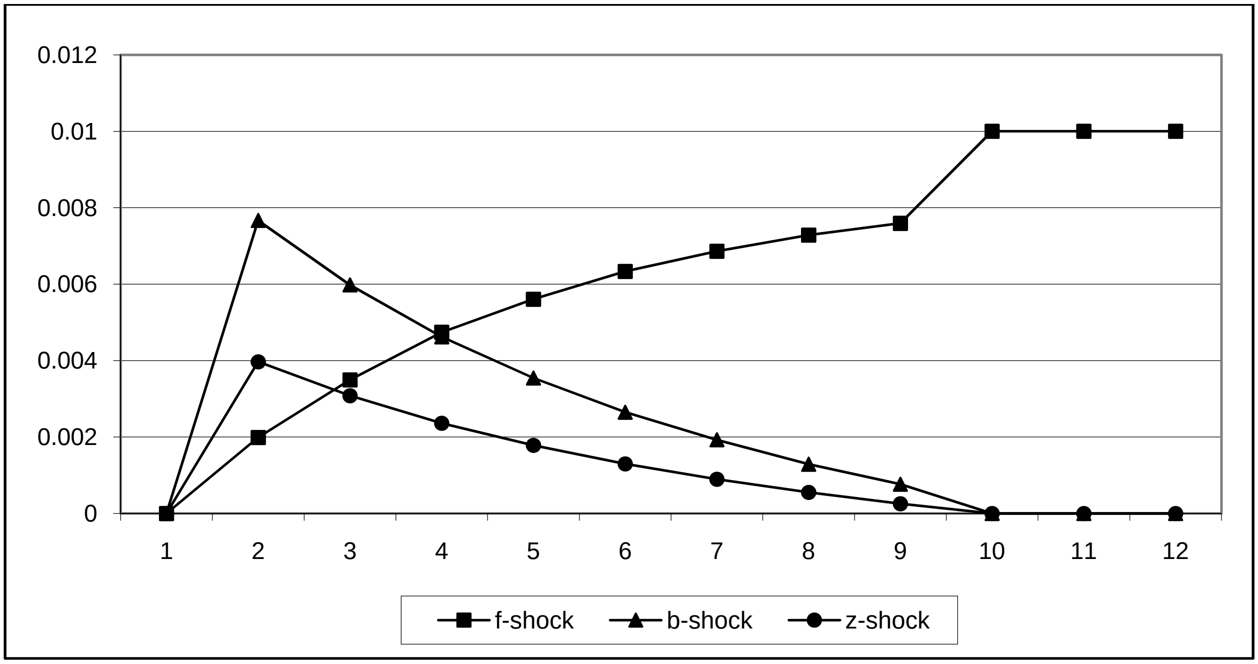

Figure 4: Impulse Response Function (ω=0.9)