MATHEMATICAL NOTATION COMPARISONS BETWEEN U.S. AND LATIN AMERICAN COUNTRIES

NUMERALS

U. S.	Latin American Countries	Descriptions
1 and 7	**1 and 7**	In many Latin American countries, the crosshatch is drawn thru the 7 to distinguish it from the numeral 1.
8	8	The numeral 8 is often drawn from the bottom up.
4	4	The numeral 4 is sometimes drawn from the bottom up. Students may confuse the 4s and the 9s.
9	**9**	The numeral 9 may resemble a lowercase “g”, particularly when written by Cuban students.

READING NUMBERS

U. S.	Latin American Countries	Descriptions
Billion = 10^9	**Billion = 10^{12}**	**READING BILLIONS** The number 23,467,891,705 is read as ▪ In U.S. as - 23billion, 467million, 891thousand, 705 ▪ In Latin American countries and in U.K. as – 23 thousand million, 467million, 891thousand, 705. In Spanish as -23mil 467millones, 891mil, 705 The number 89,520,000,000,000 is read as ▪ In U.S. as – 89trillion, 520billion ▪ In many Latin American countries and U.K. as – 89billion, 520thousand million ▪ It is noted that Brazil primarily follows the U.S. method in usage rather than their neighboring countries. ▪ As per Wikipedia at http://en.wikipedia.org/wiki/Long_and_short_scales, the U.S. uses the (short /American) scale numerical system in which every new term or period is 1,000 times greater than the previous term. A billion means a thousand millions or 10^9, a trillion means a thousand billions or 10^{12}. ▪ U.K. and some Latin American countries still use the (long/European) scale numerical system in which every term is 1,000,000 times greater than the previous terms: A billion means a million squared or 10^{12} and a trillion means a million to the 3rd power or 10^{18}.

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READING NUMBERS

U. S.	Latin American Countries	Descriptions
9,435,671	**9.435.671**	**READING NUMBERS FORM 1** ▪ In the U.S. numbers are separated by groups of 3 (otherwise known as periods) and separated by commas. ▪ In some Latin American countries, the point is used to separate such groups.
9,435,671	**9 435 671**	**READING NUMBERS FORM 2** ▪ In some Latin American countries, a space is also used to separate groups of 3 and/or periods. This is especially true in Argentina.
9,435,671	**9’435,671**	**READING NUMBERS FORM 3** ▪ As per the Secretaría de Educación Pública of Mexico 1993, millions are separated by an apostrophe, and commas separate multiples of thousands.
9,435,671	**9;435,671**	**READING NUMBERS FORM 4** ▪ The semicolon is also used in Mexico to separate the millions period from the thousands period.
- 4	**- 4 or $\overline{4}$**	**NEGATIVE NUMBERS** ▪ In Mexico negative numbers may be written either of two ways- o 1) As they are written in the U.S. with a preceding negative sign or o 2) With a bar over the number. ▪ The latter format may be confused as repeating decimal fraction.
$.\overline{3}$ **0.333…**	**$.\hat{3}$**	**REPEATING DECIMALS** ▪ In the U.S. a repeating decimal is written with a bar over the digit that is repeating and/or the repeating digit(s) are shown followed by three dots. ▪ Some books from Mexico indicate a repeating decimal with an arc rather than a line above the number.
4.56	**4,56**	**DECIMAL FRACTIONS** ▪ The POINT located at the bottom is used to define a decimal fraction. ▪ In the U.S. the point is used to separate the whole number from the fraction. ▪ In some Latin American countries, the comma is used to separate the whole number from the fraction.

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OPERATION SYMBOLS

U. S.	**Latin American Countries**	**Descriptions**
75□34 =	**75•34 =**	**MULTIPLICATION SYMBOL-FORM 1** - In the U.S. the POINT is located in the center between 2 numbers and indicates multiplication. - In Mexico, a bolder or larger raised point is used to represent multiplication.
75□34 =	**75.34 =**	**MULTIPLICATION SYMBOL-FORM 2** - In some countries, the point located on the lower part between two numbers also indicates the product of 75 and 34.
1) ÷ **2) /** **3)** $\overline{)}$	**1) ÷** **2) /** **3)** $\overline{)}$ **4) :**	**DIVISION SYMBOLS** - The Latin American countries have one additional division symbol than the U. S. It is the colon (:) Hence, the division of 26 by 2 can be written as - 26÷2, 26/2, $2\overline{)26}$ or 26**:**2.

OTHER

$\angle\alpha$ $\angle$**ABC** $\angle$**1**	$\hat{\alpha}$ $\widehat{\mathbf{ABC}}$ $\hat{\mathbf{1}}$	**ANGLE NOTATION** - Many Latin American countries place the angle symbol above the number and is also much narrower than the U. S. symbol.
May 15,2007 **5/15/2007**	**15 May, 2007** **15/5/2007**	**CALENDAR DATES** - In many Latin American countries, the month and date are reversed as compared to the format used in the U. S.

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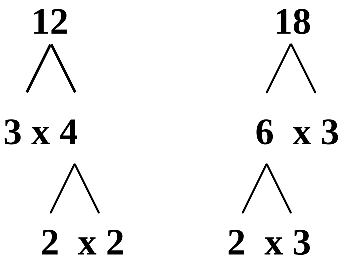
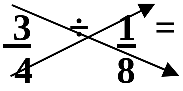
PRIME FACTORIZATION

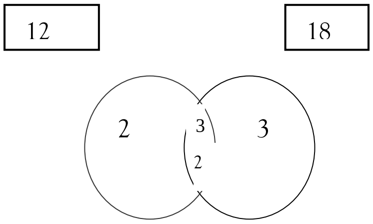
U. S.	Latin American Countries	Descriptions
 **12 = 3 x 2 x 2** **18 = 2 x 3 x 3**	$\begin{array}{r\|l} 12 & 2 \\ 6 & 2 \\ 3 & 3 \\ 1 & \end{array}$ $\begin{array}{r\|l} 18 & 2 \\ 9 & 3 \\ 3 & 3 \\ 1 & \end{array}$ **12 = 2 x 2 x 3** **18 = 2 x 3 x 3**	▪ In the U.S. finding prime factors are generally found using factor trees. Often students have difficulty finding all factors since they are spread out all over the tree. ▪ In many Latin American countries, especially in Mexico, a vertical line is used to find the same process.

DIVISION OF FRACTIONS

U. S.	Latin American Countries	Descriptions
$\frac{3}{4} \div \frac{1}{8} =$ $\frac{3}{\underset{1}{\cancel{4}}} \times \frac{\overset{2}{\cancel{8}}}{1} = 6$	 $\frac{24}{4} = 6$	▪ In the U.S. the most common procedure to divide fractions is to invert the second fraction and then multiply. ▪ In Mexico, students cross-multiply. The numerator of the first fraction is multiplied by the denominator of the 2nd fraction. That product is the numerator of the answer. Likewise, the denominator of the first fraction is multiplied by the numerator of the 2nd fraction and the product is the denominator of the answer. This is equivalent of multiplying the 1st fraction by the inverse of the 2nd fraction.

MATHEMATICAL NOTATION COMPARISONS BETWEEN U.S. AND LATIN AMERICAN COUNTRIES

Least Common Multiple

U. S.	**Latin American Countries**	▪ Descriptions
$\frac{5}{12} + \frac{3}{18} =$ **12 = 3 x 2 x 2** **18 = 2x 3 x 3** **LCM = 2^2 x 3^2** **= 4 x 9** **= 36**	$\begin{array}{cc\|l} 12 & 18 & 2 \text{ (Common factor of 12 and 18)} \\ 6 & 9 & 3 \text{ (Common factor of 12 and 18)} \\ 2 & 3 & 2 \text{ (Common factor of 12)} \\ 1 & 3 & 3 \text{ (Common factor of 18)} \end{array}$ **LCM = 2 x 3 x 2 x 3 = 36**	▪ In the U.S. the prime factorization method is one of the methods used to determine the LCM. Students find the product by using each prime the greatest number of times it appears in the factored form of any one number. ▪ To obtain common denominators, Mexican textbooks show both denominators decomposed into primes. The LCM is found by multiplying all the common prime factors and the prime factors that appear in at least one of the two denominators.
 **LCM = 2 x 3 x 2 x 3 = 36**		▪ Another way that the LCM is shown in the U.S. is using Venn diagrams as shown.

MATHEMATICAL NOTATION COMPARISONS BETWEEN U.S. AND LATIN AMERICAN COUNTRIES

Algorithms

OPERATION DESCRIPTION
SUBTRACTION

Many students come into the U.S. schools using algorithms learned in their country of origin. For example, students in many Latin American countries are taught subtraction using the **equal additions method.** According to this method the addition of equal numbers to the subtrahend and minuend does not affect the difference. To assist educators in recognizing different procedural knowledge as valid, we explain how this method works.

$\begin{array}{r} 2\,3\,5 \\ -\ \ 4\,7 \\ \hline \end{array}$ $\quad\quad$ $\begin{array}{r} 2\,3\,\overset{+10}{\boxed{5}} \\ -\ \ 4\,7 \\ \hline 8 \end{array}$	Begin the subtraction in the ones place. Since 5 is less than 7, add 10 to 5. This “changes” 5 to 15. Then subtract 15 -7= 8.
$\begin{array}{r} 2\,\overset{+100}{\boxed{3}}\,5 \\ -\ \ \overset{+10}{\boxed{4}}\,7 \\ \hline 8\,8 \end{array}$	Continue the subtraction in the tens place. To compensate for the previous addition of 10, add 10 to 40. This “changes” 40 to 50. Since 30 is less than 50, add 100 to 30. Subtract 130-50 = 80.
$\begin{array}{r} 2\,3\,5 \\ -\ \overset{+100}{\boxed{0}}\,4\,7 \\ \hline 1\,8\,8 \end{array}$	Continue the subtraction in the hundreds place. To compensate for the previous addition of 100, add 100 to 0, which is the hundreds place of 47. This “changes” 0 to 100. Finally, subtract 200-100 = 100. Answer: 188

MATHEMATICAL NOTATION COMPARISONS BETWEEN U.S. AND LATIN AMERICAN COUNTRIES

OPERATION DESCRIPTION
DIVISION

Many students come into the U.S. schools using algorithms learned in their country of origin. For example, students in many Latin American countries are expected to do and exhibit more mental computation as the following algorithm illustrates. To assist educators in recognizing different procedural knowledge as valid, we explain how this algorithm works

Format 1	**Format 2**	
$3\sqrt{74}$	$74\big\lvert\underline{3}$	In this algorithm, students will divide 3 into 74 and may write it in one of two ways.
$\begin{array}{r}2\\3\overline{)74}\\1\end{array}$	$\begin{array}{ll}74 & \big\lvert\underline{3}\\1 & \ 2\end{array}$	▪ Students typically begin to formulate and answer questions such as: How many times can 3 go into 7? Another way of asking is if we divide 70 into 3 sets, how many are in each set. ▪ Students write the 2 in the tens place, above the 7, on Format 1, but the 2 goes below the divisor when written in Format 2 style. Notice the placement of the quotient on each format. ▪ The next step is done mentally. Students multiply 3 x2 or (3 sets of 20) and then subtract. The only part that is written on paper is the remainder, 1 ten. Notice its location on both formats.
$\begin{array}{r}2\\3\overline{)74}\\14\end{array}$	$\begin{array}{ll}74 & \big\lvert\underline{3}\\14 & \ 2\end{array}$	▪ The 4 is brought down and students consider 14 next. ▪ Notice where the 14 is written on both formats.
$\begin{array}{r}24\\3\overline{)74}\\14\end{array}$	$\begin{array}{ll}74 & \big\lvert\underline{3}\\14 & \ 24\end{array}$	▪ Students now find that 3 will go into 14 three (3) times. They write 4 in the quotient’s place.
$\begin{array}{r}24\\3\overline{)74}\\14\\2\end{array}$	$\begin{array}{ll}74 & \big\lvert\underline{3}\\14 & \ 24\\\ 2 & \end{array}$	▪ Students again mentally subtract 12 from 14 and write only the remainder: 2.

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DIVISION FORMAT 3

U. S.	Latin American Countries	Descriptions
	$\begin{array}{r} 24 \\ 3\overline{)7^{1}4_{2}} \end{array}$	▪ In Mexico, this is defined as being “short division”. Notice that when 3 was divided into the 7 that produced a partial product of 2 with a remainder of 1. The remainder is placed as a superscript before the next digit in the dividend making it a 14. Again mental subtraction is done with the remainder being placed as a subscript at the end

References:

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