

Full Adder Logic

Input bit for number A	Input bit for number B	Carry bit input C_{IN}	Sum bit output S	Carry bit output C_{OUT}
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

The [full adder](#) logic relationship from the truth table at left is:

$$S = \bar{A}\bar{B}C_{IN} + \bar{A}B\bar{C}_{IN} + A\bar{B}\bar{C}_{IN} + ABC_{IN}$$

Factoring out \bar{A} and A:

$$S = \bar{A}(\bar{B}C_{IN} + B\bar{C}_{IN}) + A(\bar{B}\bar{C}_{IN} + BC_{IN})$$

which is $S = \bar{A}(B \oplus C_{IN}) + A(\overline{B \oplus C_{IN}})$

If $X = B \oplus C_{IN}$ $S = \bar{A}X + A\bar{X}$

which can be written as

$$S = A \oplus [B \oplus C_{IN}]$$

The Full Adder Carry Output:

$$C_{out} = \bar{A}B C_{in} + A\bar{B} C_{in} + AB\bar{C}_{in} + ABC_{in}$$

$$= C_{in}(\bar{A}B + A\bar{B}) + AB(\bar{C}_{in} + C_{in})$$

$$= C_{in}(A \oplus B) + AB$$