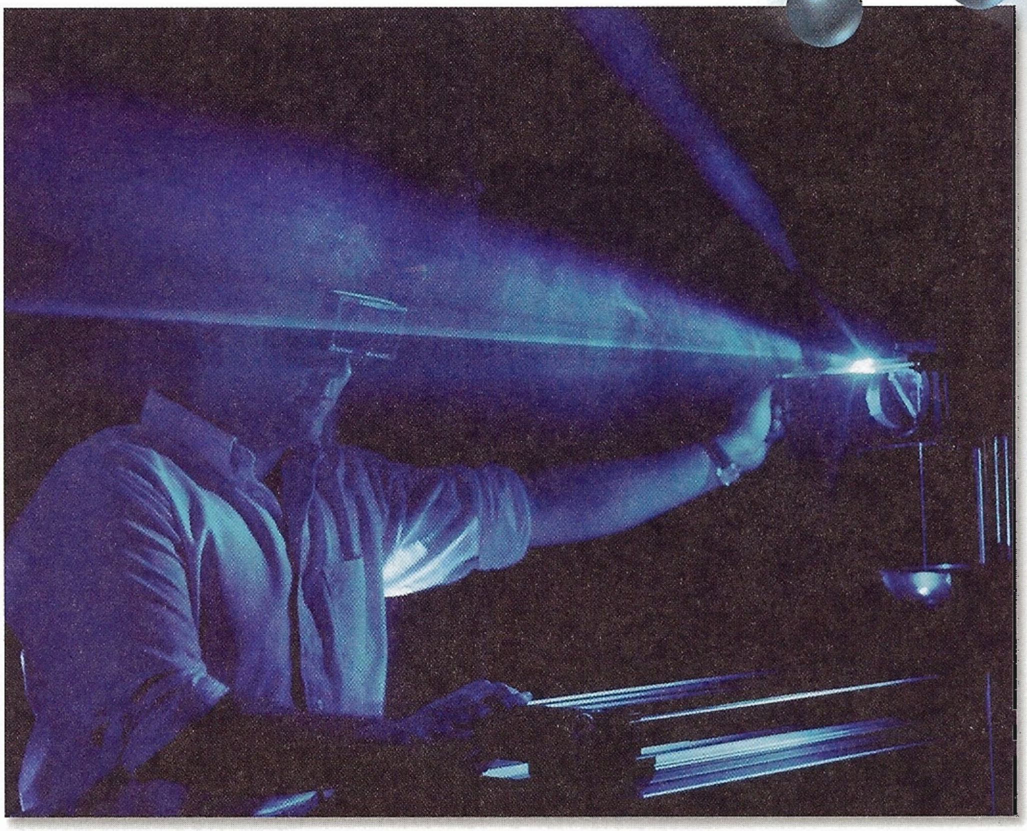
CHAPTER

2 Scientific Measurements



For very precise scientific measurements, an instrument using an argon laser beam can measure distance to one-billionth of a meter.

"Make everything as simple as possible, but not simpler."
Albert Einstein, German/Swiss/American Physicist (1879–1955)

In this chapter we will establish an important foundation for the chemical concepts and calculations discussed in later chapters. You are aware, of course, that we live in an electronic age where calculators and computers are part of our daily lives. In addition, hundreds of instruments are available that use state-of-the-art technology. In the laboratory, scientists use instruments that provide very sensitive measurements. For instance, chemists routinely use electronic balances that are so sensitive you can weigh your fingerprints!

Our discussion begins with the instruments commonly found in an introductory chemistry laboratory. Later, we will learn to add, subtract, multiply, and divide measurements obtained from these instruments. Last, and perhaps most importantly, we will learn a powerful method for solving problems in three simple steps.

2.1 UNCERTAINTY IN MEASUREMENTS

OBJECTIVES

- To identify typical instruments in a chemistry laboratory.
- To explain why instrumental measurements are never exact.

We can define a **measurement** as a number with an attached unit. For example, a 5¢ coin may have a mass of 5.005 grams. We measure the mass of the nickel with an **instrument** called a balance. The exactness of the measurement depends on the balance. For instance, electronic balances are common that measure the mass of a sample to 1/1000 of a gram.

Lab technicians routinely inject liquid samples with hypodermic syringes that measure volume to one-millionth of a liter, and electronic stopwatches are available that measure time to a nanosecond, that is, one-billionth of a second. Nevertheless, it is not possible to make exact measurements. An exact measurement is impossible because no instrument measures exactly. That is, an instrument may give a very sensitive reading, but every measurement has **uncertainty** or a degree of inexactness.

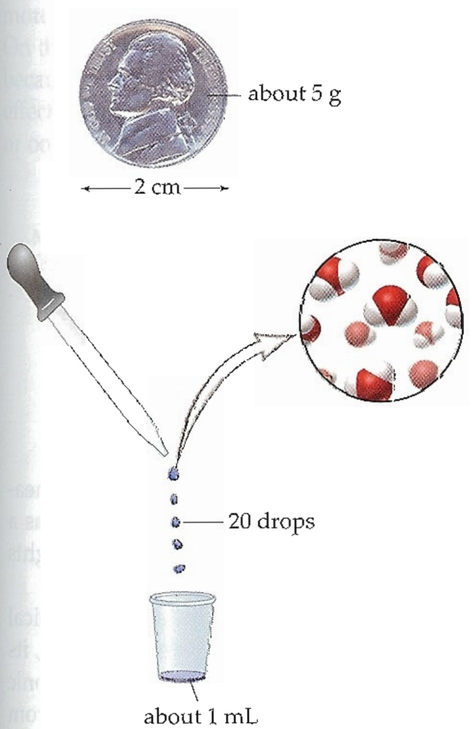
A measurement must always include a unit, such as inches, attached to a numerical value. For example, a length measurement may be 12 inches. In general, we will avoid English units such as inch (in.), pound (lb), and quart (qt), although they will be used on occasion in reference to similar metric units. In chemistry measurements, we use metric units such as **centimeter** (symbol cm), **gram** (symbol **g**), and **milliliter** (symbol **mL**).

In the metric system, a centimeter is a unit of length, a gram is a unit of mass, and a milliliter is a unit of volume. For reference, it is interesting to note that a 5¢ coin has a diameter of about 2 cm and a mass of about 5 g. Twenty drops from an eyedropper is approximately 1 mL. Figure 2.1 offers some common references for the estimation of length, mass, and volume.

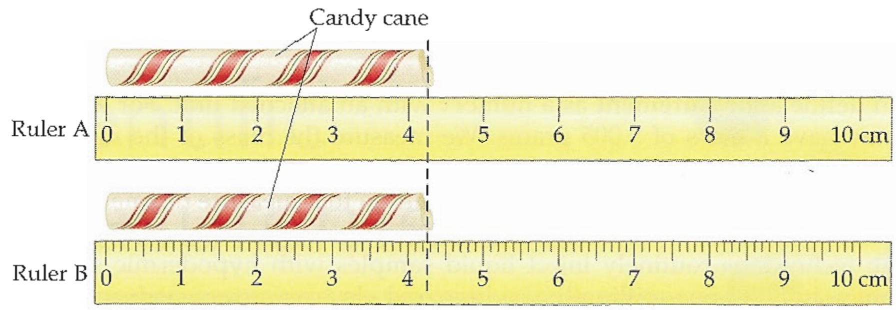
Length Measurements

To help you understand uncertainty, suppose we measure a candy cane. We have two metric rulers available that differ as shown in Figure 2.2. Both rulers are satisfactory for the task. However, Ruler B provides a more exact measurement than Ruler A.

Notice that Ruler A has ten 1 cm divisions. Since the divisions are large, we can imagine ten subdivisions. Thus, we can estimate to one-tenth of a division, that is, ±0.1 cm. On Ruler A, we see that the candy cane measures about 4.2 cm. Since the uncertainty is ±0.1 cm, a reading of 4.1 cm or 4.3 cm is also acceptable.



◀ **Figure 2.1 Estimation of Length, Mass, and Volume** The diameter of a 5¢ coin is about 2 cm, and its mass is about 5 g. The volume of 20 drops from an eyedropper is about 1 mL.



▶ **Figure 2.2 Metric Rulers for Measuring Length** On Ruler A, each division is 1 cm. On Ruler B, each division is 1 cm and each subdivision is 0.1 cm.

Notice that Ruler B has ten 1 cm divisions and ten 0.1 cm subdivisions. On Ruler B, the subdivisions are smaller. So, with Ruler B we can estimate to one-half of a subdivision, that is, we can estimate to ±0.05 cm. On Ruler B, we see that the candy cane measures about 4.25 cm. Since the uncertainty is ±0.05 cm, a reading of 4.20 cm or 4.30 cm is also acceptable.

We can compare the length of the candy cane measured with Rulers A and B as follows:

Ruler A: 4.2 ± 0.1 cm **Ruler B:** 4.25 ± 0.05 cm

In summary, Ruler A has more uncertainty and gives less precise measurements. Conversely, Ruler B has less uncertainty and gives more precise measurements. Example Exercise 2.1 further illustrates the uncertainty in recorded measurements.

Example Exercise 2.1 Uncertainty in Measurement

Which measurements are consistent with the metric rulers shown in Figure 2.2?
(a) Ruler A: 2 cm, 2.0 cm, 2.05 cm, 2.5 cm, 2.50 cm
(b) Ruler B: 3.0 cm, 3.3 cm, 3.33 cm, 3.35 cm, 3.50 cm

Solution

Ruler A has an uncertainty of ±0.1 cm, and Ruler B has an uncertainty of ±0.05 cm. Thus,
(a) Ruler A can give the measurements 2.0 cm and 2.5 cm.
(b) Ruler B can give the measurements 3.35 cm and 3.50 cm.

Practice Exercise

Which measurements are consistent with the metric rulers shown in Figure 2.2?
(a) Ruler A: 1.5 cm, 1.50 cm, 1.55 cm, 1.6 cm, 2.00 cm
(b) Ruler B: 0.5 cm, 0.50 cm, 0.055 cm, 0.75 cm, 0.100 cm

Answers: (a) 1.5 cm, 1.6 cm; (b) 0.50 cm, 0.75 cm

Concept Exercise

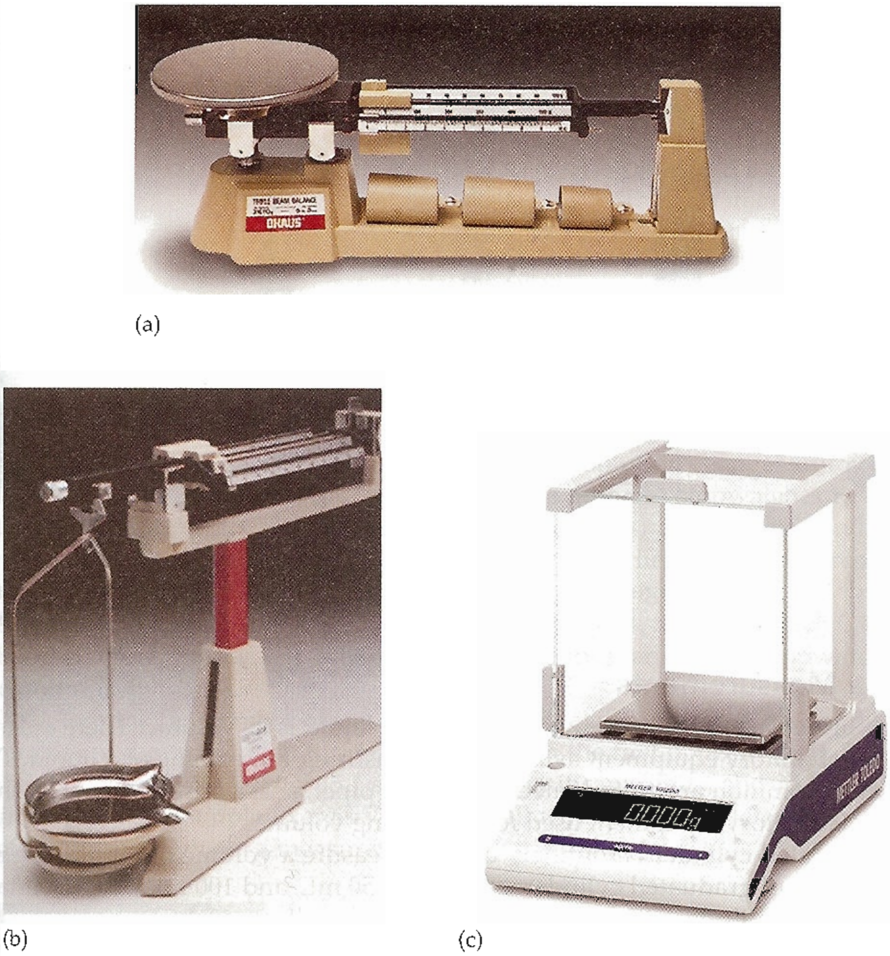
What high-tech instrument is capable of making an exact measurement?

Answer: See Appendix G.

Mass Measurements

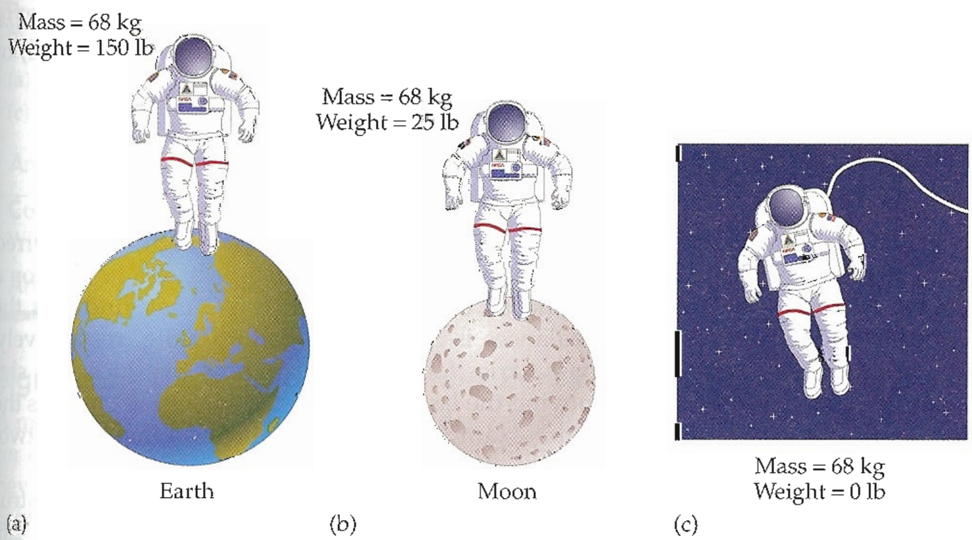
The **mass** of an object is a measure of the amount of matter it possesses. Mass is measured with a balance and is not affected by gravity. You can think of a balance as a teeter-totter, or seesaw, with two pans. After an object is placed on one pan, weights are added onto the other pan until the balance is level.

The measurement of mass has uncertainty and varies with the balance. A typical mechanical balance in a laboratory may weigh a sample to 1/100 of a gram. Thus, its mass measurements have an uncertainty of ±0.01 g. Many laboratories have electronic balances with digital displays. These balances may have uncertainties ranging from ±0.1 g to ±0.0001 g. Figure 2.3 shows three common types of balances.

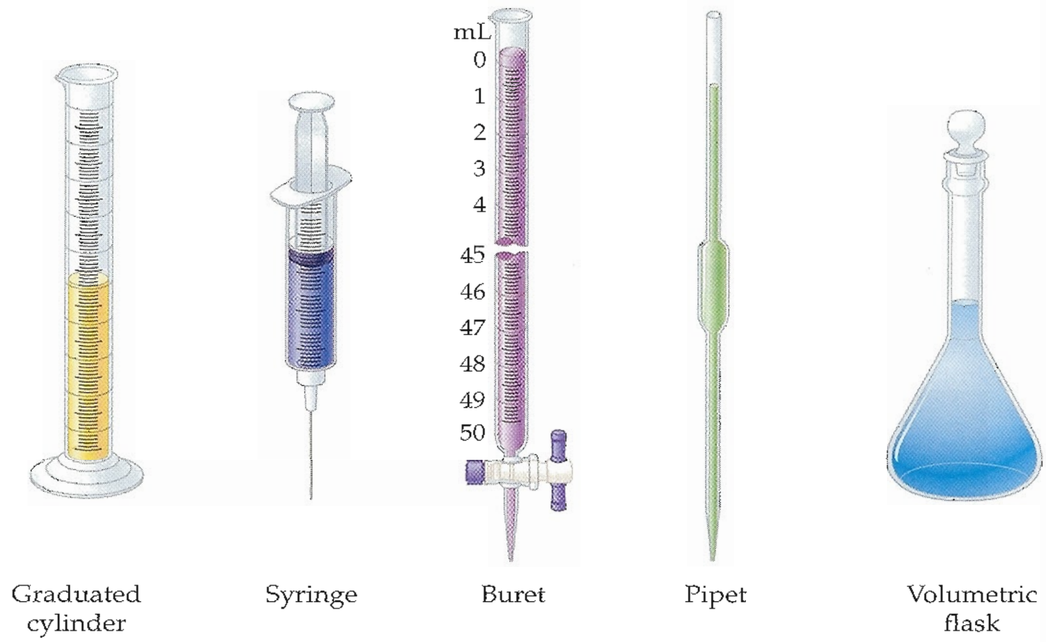


◀ **Figure 2.3 Balances for Measuring Mass** (a) A platform balance having an uncertainty of ±0.1 g. (b) A beam balance having an uncertainty of ±0.01 g. (c) A digital electronic balance having an uncertainty of ±0.001 g.

Although the term "weight" is often used to mean "mass," strictly speaking, the two terms are not interchangeable. **Weight** is the force exerted by gravity on an object. Since Earth is heavier than the Moon, Earth's gravity is greater and objects weigh more. Similarly, the same object weighs more on the huge planet Jupiter than on Earth. On the other hand, the mass of an object obtained using a balance is constant. This is because gravity operates equally on both pans of the balance, thereby canceling its effect. The mass of an object is constant whether it is measured on Earth, on the Moon, or on any other planet. Figure 2.4 illustrates the distinction between mass and weight.



◀ **Figure 2.4 Mass versus Weight** Weight is affected by gravity, whereas mass is not. (a) On Earth, the astronaut has a mass of 68 kilograms (kg) and a weight of 150 pounds (lb). (b) On the Moon, the mass remains 68 kg, but the weight is only 25 lb. (c) In space, the mass is still 68 kg although the astronaut is weightless.



▶ Figure 2.5 Instruments for Measuring Volume A graduated cylinder, a syringe, and a buret are calibrated to measure a variable quantity of liquid, whereas a volumetric pipet and a volumetric flask measure only fixed quantities, for example, 10 mL and 250 mL.

Volume Measurements

The amount of space occupied by a solid, gas, or liquid is its volume. There are many pieces of laboratory equipment available for measuring the volume of a liquid. Three of the most common are a graduated cylinder, a pipet, and a buret. Figure 2.5 shows common laboratory equipment used for measuring volume.

A graduated cylinder is routinely used to measure a volume of liquid. The most common sizes of graduated cylinders are 10 mL, 50 mL, and 100 mL. The uncertainty of a graduated cylinder measurement varies, but usually ranges from 1/10 to 1/2 of a milliliter (±0.1 mL to ±0.5 mL).

There are many types of pipets. The volumetric pipet shown in Figure 2.5 is used to deliver a fixed volume of liquid. The liquid is drawn up until it reaches a calibration line etched on the pipet. The tip of the pipet is then placed in a container, and the liquid is allowed to drain from the pipet. The volume delivered varies, but the uncertainty usually ranges from 1/10 to 1/100 of a milliliter. For instance, a 10 mL pipet can deliver 10.0 mL (±0.1 mL) or 10.00 mL (±0.01 mL), depending on the uncertainty of the instrument.

A buret is a long, narrow piece of calibrated glass tubing with a valve called a "stopcock" at the bottom end. The flow of liquid is regulated by opening and closing the stopcock, and the initial and final liquid levels in the buret are observed and recorded. The volume delivered is found by subtracting the initial buret reading from the final buret reading. Burets usually have uncertainties ranging from 1/10 to 1/100 of a milliliter. For instance, the liquid level in a buret can read 22.5 mL (±0.1 mL) or 22.55 mL (±0.01 mL), depending on the uncertainty of the instrument.

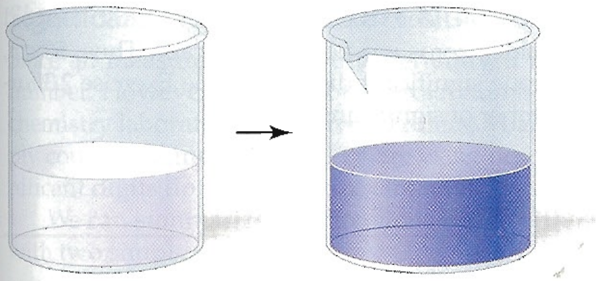
2.2 SIGNIFICANT DIGITS

OBJECTIVE

▶ To identify the number of significant digits in a given measurement.

In a properly recorded measurement, each number is a **significant digit**, also referred to as a "significant figure." For instance, suppose we find the mass of a 5¢ coin on a platform balance, a beam balance, and an electronic balance. We may find that the mass of the coin on the different balances is 5.0 g, 5.00 g, and 5.000 g, respectively. Although the uncertainty of the mass varies for the three balances, every digit is significant in each measurement. Removing the last digit from any weighing changes the uncertainty of the measurement. In this example, the measurements of mass have two, three, and four significant digits, respectively.

In every measurement, the significant digits express the uncertainty of the instrument. By way of example, let's examine the chemical reaction shown in Figure 2.6.



Colorless Solution		Blue Solution
Stopwatch A:	0 s	35 s
Stopwatch B:	0.0 s	35.1 s
Stopwatch C:	0.00 s	35.08 s

◀ **Figure 2.6 Significant Digits and a Timed Reaction** The data from the timed reaction demonstrates the uncertainty of three different stopwatches. Although each of the measurements is correct, Stopwatch C has the least uncertainty.

This is called a "clock reaction," and we note that the solution changes from colorless to blue after about 35 seconds (s). We can use three different stopwatches to time the reaction. Since a stopwatch can be calibrated in seconds (±1 s), tenths of a second (±0.1 s), or hundredths of a second (±0.01 s), we can use stopwatches having different uncertainties to time the reaction.

Stopwatch A displays 35 s, Stopwatch B displays 35.1 s, and Stopwatch C displays 35.08 s. Therefore, Stopwatch A has more uncertainty than B, and Stopwatch B has more uncertainty than C.

To determine the number of significant digits in a measurement, we simply count the number of digits from left to right, starting with the first nonzero digit. Therefore, 35 s has two significant digits, 35.1 s has three significant digits, and 35.08 s has four significant digits. Example Exercise 2.2 further illustrates how to determine the number of significant digits in a measurement.

Example Exercise 2.2 Significant Digits

State the number of significant digits in the following measurements:

(a) 12,345 cm (b) 0.123 g
(c) 0.5 mL (d) 102.0 s

Solution

In each example, we simply count the number of digits. Thus,

(a) 5 (b) 3
(c) 1 (d) 4

Notice that the leading zero in (b) and (c) is not part of the measurement but is inserted to call attention to the decimal point that follows.

Practice Exercise

State the number of significant digits in the following measurements:

(a) 2005 cm (b) 25.000 g
(c) 25.0 mL (d) 0.25 s

Answers: (a) 4; (b) 5; (c) 3; (d) 2

Concept Exercise

What type of measurement is exact?

Answer: See Appendix G.

Significant Digits and Placeholder Zeros

A measurement may contain placeholder zeros to properly locate the decimal point, for example, 500 cm and 0.005 cm. If the number is less than 1, a placeholder zero is never significant. Thus, 0.5 cm, 0.05 cm, and 0.005 cm each contain only one significant digit.

If the number is greater than 1, a placeholder zero is usually not significant. To avoid confusion, we will assume that placeholder zeros are never significant. Thus, 50 cm, 500 cm, and 5000 cm each contain only one significant digit. Example Exercise 2.3 further illustrates how to determine the number of significant digits.

Example Exercise 2.3 Significant Digits

State the number of significant digits in the following measurements:

(a) 0.025 cm (b) 0.2050 g
(c) 25.0 mL (d) 2500 s

Solution

In each example, we count the number of significant digits and disregard placeholder zeros. Thus,

(a) 2 (b) 4
(c) 3 (d) 2

Practice Exercise

State the number of significant digits in the following measurements:

(a) 0.050 cm (b) 0.0250 g
(c) 50.00 mL (d) 1000 s

Answers: (a) 2; (b) 3; (c) 4; (d) 1

Concept Exercise

What type of measurement has infinite significant digits?

Answer: See Appendix G.

Note If a placeholder zero is significant, we can express the number using a power of 10. For example, if one zero is significant in 100 cm, we can express the measurement as 1.0×10^2 cm. If both zeros are significant, we can write 1.00×10^2 cm. If neither zero is significant, we can write 1×10^2 cm. The power of 10 does not effect the number of significant digits; thus, 1.1×10^5 cm has two significant digits, and 1.11×10^{-5} cm has three significant digits. (Refer to Sections 2.6 and 2.7 for a discussion of exponents and scientific notation.)

▶ **Exact Numbers** We count seven coins in the photo, which is an exact number. This is not a measurement; thus, the number of significant digits is infinite.

Significant Digits and Exact Numbers

Since all measurements have uncertainty, a measurement never represents an exact number. However, we can obtain exact numbers when counting items. For instance, a chemistry laboratory may have 30 rulers, 3 balances, and 24 pipets. Since we have simply counted items, 30, 3, and 24 are exact numbers. As we soon learn, the rules of significant digits do not apply to exact numbers; they apply only to measurements.

We can summarize the directions for determining the number of significant digits with two simple rules.

Determining Significant Digits

Rule 1: Count the number of digits in a measurement from left to right.
  (a) Start with the first nonzero digit.
  (b) Do not count placeholder zeros (0.011, 0.00011, and 11,000 each have two significant digits).

Rule 2: The rules for significant digits apply only to measurements and not to exact numbers.

2.3 ROUNDING OFF NONSIGNIFICANT DIGITS

OBJECTIVE

▸ To round off a given value to a stated number of significant digits.

All digits in a correctly recorded measurement, except placeholder zeros, are significant. However, we often generate **nonsignificant digits** when using a calculator. These nonsignificant digits should not be reported, but they frequently appear in the calculator display. Since nonsignificant digits are not justified, we must eliminate them. We get rid of nonsignificant digits through a process of **rounding off**. We round off nonsignificant digits by following three simple rules.

Rounding Off Nonsignificant Digits

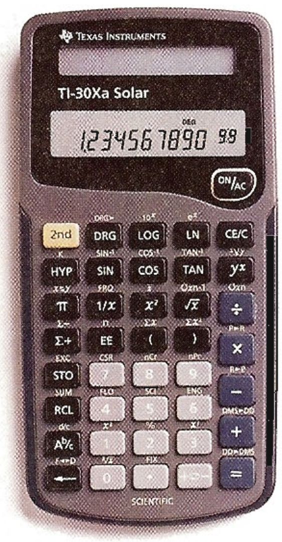
Rule 1: If the first nonsignificant digit is less than 5, drop all nonsignificant digits.

Rule 2: If the first nonsignificant digit is greater than 5, or equal to 5, increase the last significant digit by 1 and drop all nonsignificant digits.*

Rule 3: If a calculation has several multiplication or division operations, retain nonsignificant digits in your calculator display until the last operation. Not only is it more convenient, it is also more accurate.

* If the nonsignificant digit is 5, or 5 followed by zeros, an odd–even rule can be applied. That is, if the last significant digit is odd, round up; if it is even, drop the nonsignificant digits.

If a calculator displays 12.846239 and three significant digits are justified, we must round off. Since the first nonsignificant digit is 4 in 12.846239, we follow Rule 1, drop the nonsignificant digits, and round to 12.8. If a calculator displays 12.856239 and three significant digits are justified, we follow Rule 2. In this case, since the first nonsignificant digit is 5 in 12.856239, we round to 12.9.



▲ **Scientific Calculator** A calculator display often shows nonsignificant digits, which must be rounded off.

Rounding Off and Placeholder Zeros

On occasion, rounding off can create a problem. For example, if we round off 151 to two significant digits, we obtain 15. Since 15 is only a fraction of the original value, we must insert a placeholder zero; thus, rounding off 151 to two significant digits gives 150. Similarly, rounding off 1514 to two significant digits gives 1500 or 1.5×10^3. Example Exercise 2.4 further illustrates how to round off numbers.

Example Exercise 2.4 Rounding Off

Round off the following numbers to three significant digits:

(a) 22.250 (b) 0.34548
(c) 0.072038 (d) 12,267

Solution

To locate the first nonsignificant digit, count three digits from left to right. If the first nonsignificant digit is less than 5, drop all nonsignificant digits. If the first nonsignificant digit is 5 or greater, add 1 to the last significant digit.

(a) 22.3 (Rule 2) (b) 0.345 (Rule 1)
(c) 0.0720 (Rule 1) (d) 12,300 (Rule 2)

In (d), notice that two placeholder zeros must be added to 123 to obtain the correct decimal place.

Practice Exercise

Round off the following numbers to three significant digits:

(a) 12.514748 (b) 0.6015261
(c) 192.49032 (d) 14652.832

Answers: (a) 12.5 (Rule 1); (b) 0.602 (Rule 2); (c) 192 (Rule 1); (d) 14,700 (Rule 2)

Concept Exercise

How many significant digits are in the exact number 155?

Answer: See Appendix G.

2.4 ADDING AND SUBTRACTING MEASUREMENTS

OBJECTIVE

- To add and subtract measurements and round off the answer to the proper significant digits.

When adding or subtracting measurements, *the answer is limited by the value with the most uncertainty;* that is, the answer is limited by the decimal place. Note the decimal place in the following examples:

$$\begin{array}{r} 5\ \ \ \ \ \text{g} \\ 5.0\ \ \text{g} \\ +\ 5.00\ \text{g} \\ \hline 15.00\ \text{g} \end{array}$$

The mass of 5 g has the most uncertainty because it measures only ±1 g. Thus, the sum should be limited to the nearest gram. If we round off the answer to the proper significant digit, the correct answer is **15 g**. In addition and subtraction, the unit (cm, g, mL) in the answer is the same as the unit in each piece of data. Example Exercise 2.5 illustrates the addition and subtraction of measurements.

Example Exercise 2.5 Addition/Subtraction and Rounding Off

Add or subtract the following measurements and round off your answer:

(a) 106.7 g + 0.25 g + 0.195 g (b) 35.45 mL − 30.5 mL

Solution

In addition or subtraction operations, the answer is limited by the measurement with the most uncertainty.

(a) Let's align the decimal places and perform the addition.

$$\begin{array}{r} 106.7\ \ \ \ g \\ 0.25\ \ g \\ +\ \ 0.195\ g \\ \hline 107.145\ g \end{array}$$

Since 106.7 g has the most uncertainty (±0.1 g), the answer rounds off to one decimal place. The correct answer is **107.1 g** and is read "**one hundred and seven point one grams.**"

(b) Let's align the decimal places and perform the subtraction.

$$\begin{array}{r} 35.45\ mL \\ -\ 30.5\ \ mL \\ \hline 4.95\ mL \end{array}$$

Since 30.5 mL has the most uncertainty (±0.1 mL), we round off to one decimal place. The answer is **5.0 mL** and is read "**five point zero milliliters.**"

Practice Exercise

Add or subtract the following measurements and round off your answer:

(a) 8.6 cm + 50.05 cm (b) 34.1 s − 0.55 s

Answers: (a) 58.7 cm; (b) 33.6 s

Concept Exercise

When adding or subtracting measurements, which measurement in a set of data limits the answer?

Answer: See Appendix G.

2.5 MULTIPLYING AND DIVIDING MEASUREMENTS

OBJECTIVE

▶ To multiply and divide measurements and round off the answer to the proper significant digits.

Significant digits are treated differently in multiplication and division than in addition and subtraction. In multiplication and division, *the answer is limited by the measurement with the least number of significant digits.* Let's multiply the following length measurements:

$$5.15\ cm \times 2.3\ cm = 11.845\ cm^2$$

The measurement of 5.15 cm has three significant digits, and 2.3 cm has two. Thus, the product should be limited to two digits. When we round off to the proper number of significant digits, the correct answer is **12 cm^2**. Notice that the units must also be multiplied together, which we have indicated by the superscript 2. Example Exercise 2.6 illustrates the multiplication and division of measurements.

Example Exercise 2.6 Multiplication/Division and Rounding Off

Multiply or divide the following measurements and round off your answer:

(a) 50.5 cm × 12 cm (b) 103.37 g/20.5 mL

Solution

In multiplication and division operations, the answer is limited by the measurement with the least number of significant digits.

(a) In this example, 50.5 cm has three significant digits and 12 cm has two.

$$(50.5 \text{ cm})\,(12 \text{ cm}) = 606 \text{ cm}^2$$

The answer is limited to two significant digits and rounds off to **610 cm²** after inserting a placeholder zero. The answer is read "**six hundred and ten square centimeters**."

(b) In this example, 103.37 g has five significant digits and 20.5 mL has three.

$$\frac{103.37 \text{ g}}{20.5 \text{ mL}} = 5.0424 \text{ g/mL}$$

The answer is limited to three significant digits and rounds off to **5.04 g/mL**. Notice that the unit is a ratio; the answer is read as "**five point zero four grams per milliliter**."

Practice Exercise

Multiply or divide the following measurements and round off your answer.

(a) (359 cm) (0.20 cm) (b) 73.950 g/25.5 mL

Answers: (a) 72 cm^2; (b) 2.90 g/mL

Concept Exercise

When multiplying or dividing measurements, which measurement in a set of data limits the answer?

Answer: See Appendix G.

OBJECTIVES

- To explain the concept of exponents and specifically powers of 10.
- To express a value as a power of 10 and as an ordinary number.

2.6 EXPONENTIAL NUMBERS

When a value is multiplied times itself, the process is indicated by a number written as a superscript. The superscript indicates the number of times the process is repeated. For example, if the number 2 is multiplied two times, the product is expressed as 2^2. Thus, (2) (2) = 2^2. If the number 2 is multiplied three times, the product is expressed as 2^3. Thus, (2) (2) (2) = 2^3.

A superscript number indicating that a value is multiplied times itself is called an **exponent**. If 2 has the exponent 2, the value 2^2 is read as "2 to the second power" or "2 squared." The value 2^3 is read as "2 to the third power" or "2 cubed."

Powers of 10

A **power of 10** is a number that results when 10 is raised to an exponential power. You know that an exponent raises any number to a higher power, but we are most interested in the base number 10. A power of 10 has the general form

exponent

$$10^n$$

base number

The number 10 raised to the n power is equal to 10 multiplied times itself n times. For instance, 10 to the second power (10^2) is equal to 10 times 10. When we write 10^2 as an ordinary number, we have 100. Notice that the exponent 2 corresponds to the number of zeros in 100. Similarly, 10^3 has three zeros (1000) and 10^6 has six zeros (1,000,000).

The exponent is positive for all numbers greater than 1. Conversely, the exponent is negative for numbers less than 1. For example, 10 to the negative first power (10^{-1}) is equal to 0.1, 10 to the negative second power (10^{-2}) is equal to 0.01, and 10 to the negative third power (10^{-3}) is equal to 0.001. Table 2.1 lists some powers of 10 along with the equivalent ordinary number.

Although you can easily carry out operations with exponents using an inexpensive scientific calculator, you will have greater confidence if you understand exponents. Example Exercises 2.7 and 2.8 further illustrate the relationship between ordinary numbers and exponential numbers.