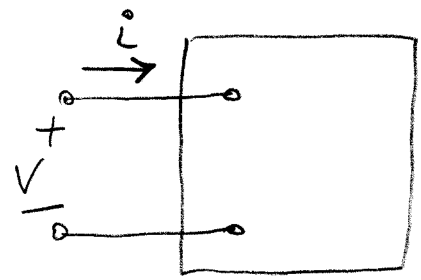
Engr 17 Final Review

Chapter 1 – circuit Variables

Passive sign convention



$$\text{Power} = \frac{dw}{dt} = v\,i$$

$P > 0$	Power absorbed by the circuit
$P < 0$	Power delivered by the circuit

Chapter 2 – Circuit Elements

v_s Ideal Voltage source – maintains voltage regardless of current in (thru) the device.

i_s Ideal current source – maintains current regardless of the voltage across the device

Dependent sources – value determined by some other source

Ohm's Law $\quad V = iR$

$$P = vi = i^2R = \frac{v^2}{R}$$

Node – 2 or more circuit elements join

Kirchoff's current law $\sum_{node} i = 0$ at any node

Kirchoffs voltage law $\sum_{loop} v = 0$ around any closed path

Chapter 3 - Simple Resistive circuits

"Resistors add in series"

$$R_{eq} = R_1 + R_2 + \cdots R_k$$

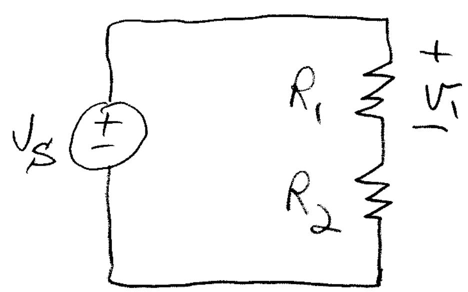
Parallel R_{eq}

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_k}$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$ Two resistors in parallel

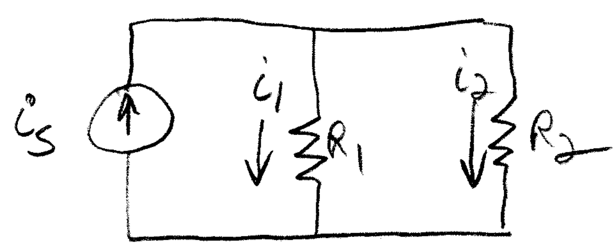
Voltage Divider

$$v_1 = \frac{R_1}{R_1 + R_2} V_s$$



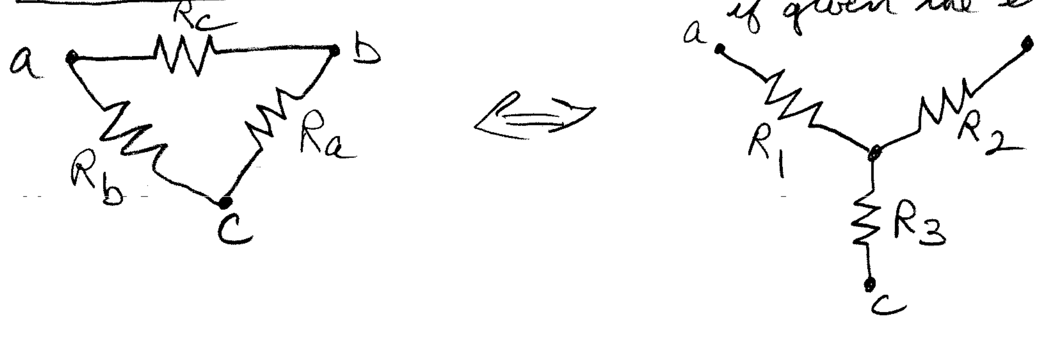
Current Divider

$$i_1 = \frac{R_2}{R_1 + R_2} i_s$$



Delta to Wye (or Pi to Tee)

Be able to solve transform if given the eqns.



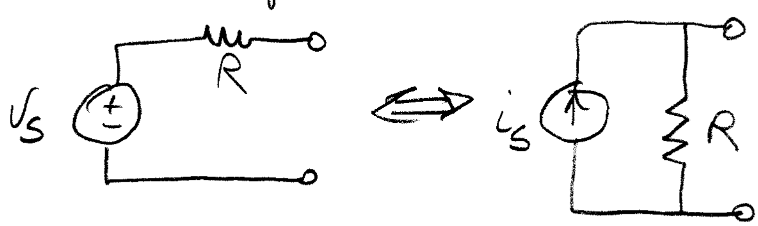
Chap 4 – Techniques of Circuit Analysis

Know how to apply the definitions of node, essential node, path, branch, essential branch, and mesh

Node-Voltage – choose reference node & write current in/out of the node in terms of the voltage variables $\left(\frac{V_1 - V_2}{R}\right)$

Mesh-Current → assign reference currents in a loop. Write the sum of the voltages (current × resistance) around the loop = zero.

Source Transform

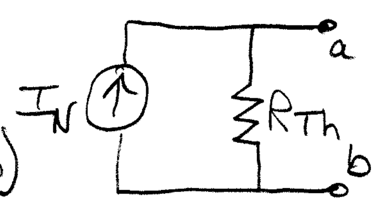


$V_s = i_s R$

Thevenin Equivalent – simplify circuit to

$V_{Th} = V_{open\ circuit}$ (at terminal ab)

Norton Equivalent – simplify to



$I_N = I_{short\ circuit}$ (across terminal ab)

$$R_{Th} = \frac{V_{Th}}{I_{SC}}$$

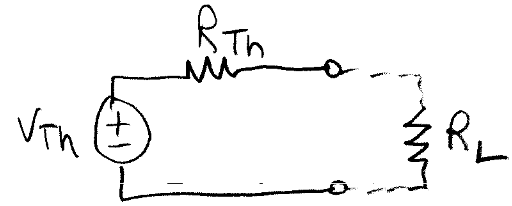
we possible (usually simpler) find R_{Th} by deactivating the Independent sources & then apply series or parallel combinations to find R_{eq}

When Dependent Sources are in the circuit - you must apply a "test" voltage V_T, find current I_T (by circuit analysis)

$$R_{Th} = \frac{V_T}{I_T}$$

The trick is to restate the dependent source in terms of V_T & I_T.

Max Power



max when $R_{Load} = R_{Th}$

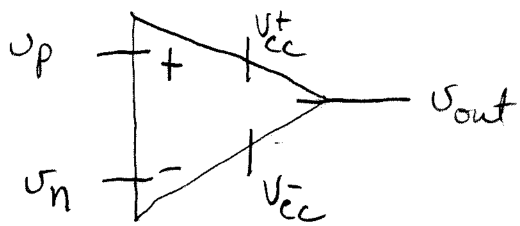
$$p = \frac{V_{Th}^2}{4 R_L}$$

Superposition → for a linear system, you can find the result (v or i) for each independent source individually & then sum all the results into the complete answer.

[All equations marked with * will be given to you on the test]

Chapter 5 - Ideal Op-Amp

sec 5.1 thru 5.4
(5.6 & 5.7 not covered)



V_{out} must be $\leq$ supply voltage

$$v_o = \begin{cases} -V_{cc} \\ A(v_p - v_n) \\ +V_{cc} \end{cases}$$

$v_p = v_n$

$i_p = i_n = 0$

Be able to use node-voltage analysis to solve for

$v_{out} = k\, v_{in}$ where k = scaling factor

Inverting Amplifier $v_o = \dfrac{-R_f}{R_s} v_{in}$

Summing Amplifier $v_o = -\left(\dfrac{R_f}{R_a} v_a + \dfrac{R_f}{R_b} v_b + \dfrac{R_f}{R_c} v_c\right)$

non-inverting Amplifier $v_o = \dfrac{R_s + R_f}{R_s} v_{in}$

Examine the circuits in the chapter

Chapter 6 – Sec 6.1 thru 6.3
(Sec 6.4 & 6.5 not covered)

Inductance – relates induced voltage (induced by a time varying magnetic field) to the current.

capacitance – relates displacement current (produced by a time-varying electric field) to the voltage.

Inductor	Capacitor
$v = L\dfrac{di}{dt}$	$v = \dfrac{1}{C}\int_{t_0}^{t} i\, d\tau + v(0)$
$i(t) = \dfrac{1}{L}\int_{t_0}^{t} v\, d\tau + i(t_0)$	$i = C\dfrac{dv}{dt}$
power* $p = Li\dfrac{di}{dt}$	* $p = Cv\dfrac{dv}{dt}$
energy $w = \frac{1}{2}Li^2$	$w = \frac{1}{2}Cv^2$
Inductors add in series	capacitors add in parallel
$L_{eq} = L_1 + L_2 + \cdots + L_n$ (series)	$\dfrac{1}{C_{eq}} = \dfrac{1}{C_1} + \dfrac{1}{C_2} + \cdots + \dfrac{1}{C_n}$ (series)
$\dfrac{1}{L_{eq}} = \dfrac{1}{L_1} + \dfrac{1}{L_2} + \cdots + \dfrac{1}{L_n}$ (parallel)	$C_{eq} = C_1 + C_2 + \cdots + C_n$ (Parallel)

Chapter 7 - Response of 1st order RL and RC circuits

Natural Response - no source but can have initial energy.

Step Response - sudden onset of constant voltage or current.

RL circuit (Natural Response)

$$\left.\begin{aligned} i(t) &= I_0\, e^{-\frac{R}{L}t} \\ v(t) &= I_0 R\, e^{-\frac{R}{L}t} \end{aligned}\right\} \text{for } t \geq 0 \qquad \tau = \frac{L}{R}$$

*power $= vi = i^2 R = \dfrac{v^2}{R} = I_0^2 R\, e^{-2\frac{t}{\tau}}$ $\quad t \geq 0^+$

*energy $w = \frac{1}{2} L I_0^2 (1 - e^{-2\frac{t}{\tau}})$ $\quad t \geq 0$

Steady state $\stackrel{d}{=}$ 5 time constants

RC circuit (Natural Response)

$v(t) = V_0 e^{-\frac{t}{\tau}}$ $\quad t \geq 0$ $\qquad \tau = RC$

$i(t) = \dfrac{V_0}{R} e^{-\frac{t}{\tau}}$ $\quad t \geq 0^+$ $\qquad V_0 =$ initial capacitor voltage

$*P = \dfrac{V_0^2}{R} e^{-2\frac{t}{\tau}}$ $\quad t \geq 0^+$

$*w = \frac{1}{2} C V_0^2 (1 - e^{2\frac{t}{\tau}})$ $\quad t \geq 0$

RL circuit (Step Response)

recall $V_L = L\frac{di}{dt}$

$* \; i(t) = \frac{V_S}{R} + \left(I_0 - \frac{V_S}{R}\right)e^{-\frac{t}{\tau}}$

$* \; v(t) = (V_S - I_0 R)\, e^{-\frac{t}{\tau}}$

$t \geq 0$

so derivative involved in $v(t)$

RC circuit (Step Response)

$* \; v_C(t) = I_S R + (V_0 - I_S R)\, e^{-\frac{t}{\tau}} \quad t \geq 0$

$i = C\frac{dv}{dt}$

$* \; i_C(t) = \left(I_S - \frac{V_0}{R}\right) e^{-\frac{t}{\tau}} \quad t \geq 0^+$

again derivative involved

General soln (sec 7.4) not on the test.

Sequential switching (sec 7.5) not on the test

sec 7.6 Unbounded Response

must have dependent source!

$R_{Th} \rightarrow$ Negative resistance (Thévenin resistance)

usually use "test method" to find

$\frac{v_T}{i_T} = R_{Th}$

(Hint: remember Thévenin + Norton equivalents is one of the three things you learn in this course!)

Sec 7.7 Integrating Op-Amp

see fig 7.40 on page 316

$$* \; v_o(t) = \frac{-1}{R_s C_f} \int_{t_o}^{t} v_s \, d\tau + v_o(t_o)$$

Chapter 8 N.R. & S.R. for RLC circuits - 2nd order system

Characteristic eqn. $s^2 + 2\alpha s + \omega_o^2 = 0$

roots $s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_o^2}$

$$\left.\begin{aligned} \alpha &= \frac{1}{2RC} \text{ (Parallel)} \\ &= \frac{R}{2L} \text{ (series)} \end{aligned}\right\} \text{Neper Frequency}$$

$\omega_o = \frac{1}{\sqrt{LC}}$ resonant radian frequency

overdamped $\omega_o^2 < \alpha^2$ (or $\alpha^2 > \omega_o^2$)

$s_{1,2}$ = real & distinct roots

Sluggish response

underdamped $\omega_o^2 > \alpha^2$

$s_{1,2}$ = complex conjugate pair roots $(x \pm jy)$

rapid but "ringing" response - decaying oscillations critically damped $\alpha^2 = \omega_0^2$

$s_{1,2}$ = real & repeating roots

"on the verge" of oscillating

Natural Response

Over $* \; x(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$

$* \; x(0) = A_1 + A_2$

$* \; \frac{dx(0)}{dt} = s_1 A_1 + s_2 A_2$

Under $* \; x(t) = (B_1 \cos \omega_d t + B_2 \sin \omega_d t) e^{-\alpha t}$

$* \; x(0) = B_1$

$* \; \frac{dx(0)}{dt} = -\alpha B_1 + \omega_d B_2$

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

Critical

$* \; x(t) = (D_1 t + D_2) e^{-\alpha t}$

$* \; x(0) = D_2$

$* \; \frac{dx(0)}{dt} = D_1 - \alpha D_2$

solve for voltage (or current) at once then find the current (or voltage).

Step Response

over

$*\; x(t) = x_{final} + A_1' e^{s_1 t} + A_2' e^{s_2 t}$

$*\; x(0) = x_f + A_1' + A_2'$

$*\; \dfrac{dx(0)}{dt} = A_1' s_1 + A_2' s_2$

under

$*\; x(t) = x_f + (B_1' \cos\omega_d t + B_2' \sin\omega_d t)\, e^{-\alpha t}$

$*\; x(0) = x_f + B_1'$

$*\; \dfrac{dx(0)}{dt} = -\alpha B_1' + \omega_d B_2'$

critical

$*\; x(t) = x_f + D_1' t e^{-\alpha t} + D_2' e^{-\alpha t}$

$*\; x(0) = x_f + D_2'$

$*\; \dfrac{dx(0)}{dt} = D_1' - \alpha D_2'$

Know how to use these equations!

Also, clearly relate V_{cap} to $I_{inductor}$ for all times but in particular at $t=0$ and $t=\infty$

Chapter 9 - Sinusoidal Steady-State Analysis

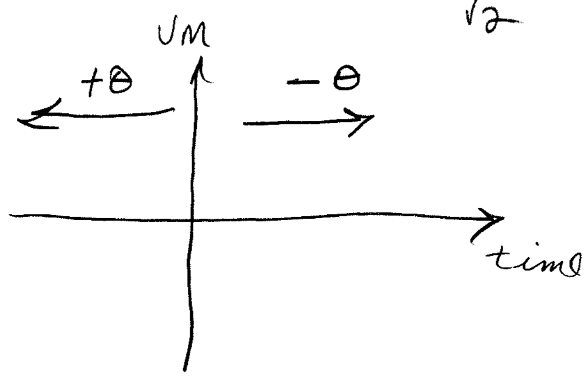
$$v(t) = V_m \cos(\omega t + \theta)$$

$$\omega = 2\pi f = \frac{2\pi}{T} \quad \text{rad/sec}$$

$$\theta^\circ \left(\frac{\pi}{180^\circ}\right) = (\ ) \text{ radians/sec}$$

$$V_{rms} = \underbrace{\sqrt{\underbrace{\frac{1}{T}\int_{t_0}^{t_0+T} \underbrace{V_m^2 \cos^2(\omega t+\theta)}_{\text{square}}\, dt}_{\text{mean over one period } T}}}_{\text{(square) root}}$$

for cos or sin $\quad V_{rms} = \frac{V_M}{\sqrt{2}}$



$$\sin(\theta) = \cos(\theta - 90^\circ)$$

Total Response = Transient + steady state

steady state solution for linear systems

1. response is a sinusoidal
2. freq remains the same.
3. all else may vary.

<u>Resistor</u> $\mathbf{V} = \mathbf{I} R$ no phase shift

<u>Inductor</u> $v = L \frac{di}{dt}$ "current lags the voltage"

$\mathbf{V} = j\omega L \, \mathbf{I}$

<u>Capacitor</u> $i = c \frac{dv}{dt}$

$\mathbf{V} = j\left(\frac{-1}{\omega c}\right) \mathbf{I}$ "current leads the voltage"

<u>Impedance</u>

$\mathbf{V} = Z \, \mathbf{I}$

$$Z = \begin{cases} R & \text{resistor} \\ j\omega L & \text{inductor} \\ j\left(\frac{-1}{\omega c}\right) & \text{capacitor} \end{cases}$$

KVL & KCL all still apply - must now solve the complex number math.

<u>Series</u>

$$Z_{eq} = Z_1 + Z_2 + \cdots + Z_k$$

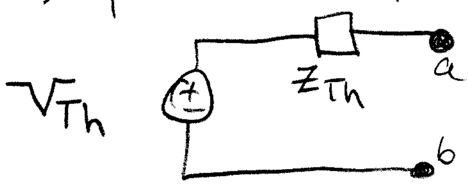
<u>Parallel</u>

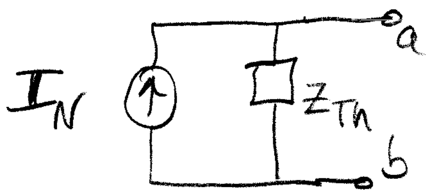
$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \cdots + \frac{1}{Z_k} \qquad Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

Δ to Y in Freq domain $\rightarrow$ not on the final

Thevenin & Norton

replace R_{Th} with Z_{Th}





Node-Voltage & Mesh as before

Be sure to solve the complex number math carefully.

Chap 10 - Sinusoidal Steady-State Power

after the trig identities we write the power as

$$P_{Total} = P + P\cos 2\omega t - Q\sin 2\omega t$$

where P = average power = $\dfrac{V_m I_m}{2}\cos(\theta_v - \theta_i)$

Q = reactive power = $\dfrac{V_m I_m}{2}\sin(\theta_v - \theta_i)$

note $\dfrac{V_m}{\sqrt{2}} = V_{rms} \stackrel{d}{=} V_{eff}$

$\dfrac{I_m}{\sqrt{2}} = I_{rms} \stackrel{d}{=} I_{eff}$

$pf = \cos(\theta_v - \theta_i)$ power factor

must state whether current is "leading" (cap) or "lagging" (inductor)

$rf = \sin(\theta_v - \theta_i)$ reactive factor

Complex Power

$\mathcal{S}$ = apparent power $= P + jQ$

$$= \frac{1}{2} \mathbf{V} \mathbf{I}^* = \mathbf{V}_{rms} \mathbf{I}^*_{rms}$$

$$= I_{rms}^2 (Z) = \frac{V_{rms}^2}{Z^*}$$

not a phasor

$|S| = \sqrt{P^2 + Q^2}$ in units of Volt·Amp

P has units of watt

Q has units of Volt·Amp·Reactive (VAR)

Max Power

$Z_L = Z^*_{Th}$ $\quad P_{max} = \frac{1}{4} \frac{|V_{Th}|^2}{R_L} = \frac{1}{8} \frac{V_m^2}{R_L}$