FE/EIT Review

Circuits

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References

John A. Camara, Electrical Engineering Reference Manual, 6th edition, Professional Publications, Inc, 2002.

John A. Camara, Practice Problems for the Electrical and Computer Engineering PE Exam, 6th edition, Professional Publications, Inc, 2002.

National Council of Examiners for Engineering & Surveying, Principles and Practice of Engineering, Electrical and Computer Engineering, Sample Questions and Solutions, NCEES, 2001.

National Council of Examiners for Engineering & Surveying, Fundamental of Engineering, Supplied-Reference Handbook, NCEES, 2008.

References

Michael A. Lindeburg, PE, FE Review Manual, Rapid Preparation for the General Fundamentals of Engineering Exam, 2nd Edition, Professional Publications, 2006.

Michael A. Lindeburg, PE, FE/EIT Sample Examinations, 2nd Edition, Professional Publications, 2006.

Introduction

The morning FE examination will have 120 questions in a 4 hour period. The questions cover 12 topic areas:

1. Mathematics
2. Engineering Probability and Statistics
3. Chemistry
4. Computers
5. Ethics and Business Practices
6. Engineering Economics
7. Engineering Mechanics (Statics and Dynamics)
8. Strength of Materials
9. Material Properties
10. Fluid Mechanics
11. Electricity and Magnetism
12. Thermodynamics

Introduction

Section XI. Electricity and Magnetism includes:

A. Charge, Energy, current, voltage, power

B. Work done in moving a charge in an electric field
relationship between voltage and work

C. Force between charges

D. Current and voltage laws
Kirchhoff's voltage law, Kirchhoff's current law, Ohm's law

E. Equivalent circuit
Series, Parallel, Thévenin/Norton equivalent

Introduction

Section XI. Electricity and Magnetism includes:

F. Capacitance and Inductance

G. Reactance and impedance, susceptance and admittance

H. AC Circuits

I. Basic complex algebra

Scope of the Systems

You will be expected to analyze Linear, Lumped parameter, time invariant systems.

Linear – response is proportional to V or I
(no higher order terms needed)

Lumped Parameter - Electrical effects happen instantaneously in the system. Low frequency or small size (about 1/10 of the wavelength).

Time Invariant - The response of the circuit does NOT depend on when the input was applied.

Units

Volt

The Potential difference is the energy required to move a unit charge (the electron) through an element (such as a resistor).

Amp

Electric current is the time rate of change of the charge, measured in amperes (A).

Ohm

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

Units

Coulomb

The amount of charge that crosses a surface in one second when a steady current of one ampere flows.

Farad

Capacitance is the ratio of the charge on one plate of a capacitor to the voltage difference between the two plates - measured in farads (F).

Henry

Inductance is the property whereby an inductor exhibits opposition to the charge of current flowing through it - measured in henrys (H).

Algebra of Complex Numbers

A complex number z, consists of the sum of real and imaginary numbers.

$$z = a \pm jb$$

The rectangular form can be found from a phasor with polar magnitude c and an angle θ.

$$a = c\cos\theta$$
$$b = c\sin\theta$$
$$z = a + jb \quad = c\cos\theta + jc\sin\theta$$

Algebra of Complex Numbers

A complex number z, consists of the sum of real and imaginary numbers.

$$z = a \pm jb$$

The phasor form (polar) can be found from the rectangular form as follows:

$$c = \sqrt{a^2 + b^2}$$
$$\theta = \tan^{-1}\left(\frac{b}{a}\right)$$
$$z = c\langle\theta \quad = \sqrt{a^2+b^2}\,\langle \tan^{-1}\left(\frac{b}{a}\right)$$

Algebra of Complex Numbers

Add or subtract complex numbers in the rectangular form.

$$z = a + jb$$
$$y = c - jd$$
$$z - y = (a-c) + j(b-d)$$

Multiply or divide complex numbers in polar form.

$$m = \frac{z\langle\theta}{y\langle\phi} \quad = \frac{z}{y}\langle(\theta-\phi)$$

Algebra of Complex Numbers

Complex numbers can also be expressed in exponential form by use of Euler's Identity.

$$e^{\pm j\theta} = \cos\theta \pm j\sin\theta$$

The trigonometric functions then become:

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

Example - Algebra of Complex Numbers

The rectangular form of a given complex number is

$$z = 3 + j4$$

What is the number when using trigonometric functions?

$$(A)\ 5e^{j36.86^\circ}$$

$$(B)\ (5)(\cos 36.86^\circ + j\sin 36.86^\circ)$$

$$(C)\ \cos 0.64 + j\sin 0.64$$

$$(D)\ (5)(\cos 0.93 + j\sin 0.93)$$

The correct answer is D.

Electrostatics

Electric charge is a fundamental property of subatomic particles.

A Coulomb equals a very large number of charged particles. This results in a Farad also being a very large number of charged particles. Thus we usually deal with very small amount of charge.

The charge of one electron is -1.602 x 10^{-19} C.

The charge of one proton is +1.602 x 10^{-19} C.

Electrostatics

An electric field E with units of volts/meter is generated in the vicinity of an electric charge.

The force applied by the electric field E is defined as the electric flux of a positive charged particle introduced into the electric field.

$$F = QE$$

The work W performed on a moving charge Q_B a certain distance in a field created by charge Q_A is given by

$$W = -Q\int_{r_1}^{r_2} E\,dL \;= -\int_{r_1}^{r_2} F\,dr \;= -\int_{r_1}^{r_2} \frac{Q_A Q_B}{4\pi\varepsilon r^2}dr \;= \frac{Q_A Q_B}{4\pi\varepsilon}\left(\frac{1}{r_2} - \frac{1}{r_1}\right)$$

Electrostatics

Work is performed only if the charges are moved closer or farther apart.

For a uniform electric field (such as inside a capacitor), the work done in moving a charge parallel to the E field is

$$W = -Q\Delta V$$

The last equation says the work done is the charge times the change in voltage the charge experienced by the movement.

The electric field strength between two parallel plates with a potential difference V and separated by a distance d is

$$E = \frac{V}{d}$$

Electrostatics

Current is the movement of charge.

By convention the current moves in a direction opposite to the flow of electrons.

Current is measured in Amperes and is the time rate of change of charge.

$$i(t) = \frac{dq(t)}{dt}$$

If the rate of change in the charge is constant, the current can be written as

$$I = \frac{dQ}{dt}$$

The above equations largely describe the behavior of a capacitor.

Magnetic Fields

A magnetic field can exist only with two opposite and equal poles.

While scientists have searched for a magnetic monopole it has not yet been found.

A magnetic field induces a force on a stationary charge.
Conversely a moving charge induces a magnetic field.

The magnetic field density is a vector quantity and given by B

$$B = \frac{\phi}{A} = \frac{Magnetic\ Flux}{Area}$$

An inductor (or transformer) relies on the magnetic field interaction with moving charges to alter the behavior of a circuit.

Practice Problem - Electrostatics

Determine the magnitude of the electric field necessary to place a 1 N force on an electron.

$$F = |-Q|E$$

Thus

$$E = \frac{F}{Q} = \frac{1N}{1.602\times10^{-19}C} = 6.24\times10^{18}\,\tfrac{N}{C}$$

$$= 6.24\times10^{18}\,\tfrac{V}{m}$$

Practice Problem - Electrostatics

A current of 10 A flows through a 1 mm diameter wire. What is the average number of electrons per second that pass through a cross section of the wire?

$(A)\ 1.6\times10^{18}\ \frac{electrons}{\sec}$

$(B)\ 6.2\times10^{18}\ \frac{electrons}{\sec}$

$(C)\ 1.6\times10^{19}\ \frac{electrons}{\sec}$

$(D)\ 6.3\times10^{19}\ \frac{electrons}{\sec}$

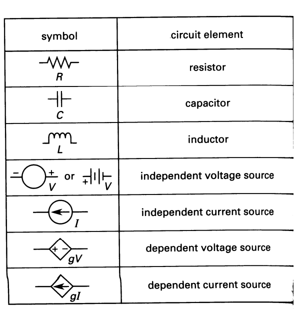
The closest answer is D.

DC Circuits

DC Circuits include the following topics:

- DC Voltage
- Resistivity
- Resistors in Series and Parallel
- Power in a Resistive Element
- Capacitors
- Inductors
- Capacitors in Series and Parallel

Circuit Symbols



DC Circuits

Electrical Circuits contain active and passive elements.

Active elements can generate electric energy – voltage sources, current sources, op-amps

Passive elements absorb or store electric energy – capacitor, inductor, resistor.

An ideal voltage source supplies power at a constant voltage regardless of the current the external circuit demands.

An ideal current source supplies power at a constant current regardless of the voltage the external circuit demands.

Dependent sources deliver voltage and current at levels determined by voltages or currents elsewhere in the circuit.

DC Voltage

Symbol: V or E (electromotive force)

Circuit usage: V
or v(t) when voltage may vary.

Voltage is a measure of the DIFFERENCE in electrical potential between two points.

Voltage ACROSS two points.

DC current

Symbol: A (Coulomb per second)

Circuit usage: I
or i(t) when current may vary with time.

Amperage is a measure of the current flow past a point.

Current THROUGH a circuit element.

Definitions

A Direct Current (dc) is a current whose polarity remains constant with time. The amplitude is usually considered to remain constant.

The amplitude is usually considered to remain constant.

An Alternating Current (ac) is a current that varies with time.

A common form of AC is the sinusoidal power delivered by the power company.

Resistivity

Symbol: R measured in ohms, Ω

Circuit usage: R

Resistance is the property of a circuit or circuit element to oppose current flow.

$$R = \rho \frac{L}{A} = (resistivity)\frac{Length}{Area}$$

A circuit with zero resistance is a *short circuit*.

A circuit with an infinite resistance is a *open circuit*.

Resistors in Series and Parallel

Series Resistors:

$$R_{eq} = R_1 + R_2 + \ldots\ldots$$

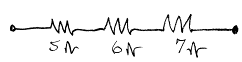
Parallel Resistors:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots\ldots$$

For Two Resistors in Parallel:

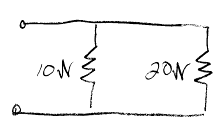
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

Example - Calculating equivalent resistance



$R_{eq} = ?$ $\quad = 5 + 6 + 7 = 18\Omega$

The equivalent resistance R_{EQ} is larger than the largest resistor.



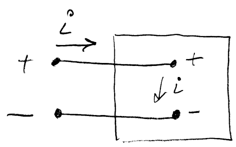
$R_{eq} = ?$ $\quad = \frac{10(20)}{10+20} = \frac{200}{30} = 6.67\Omega$

The equivalent resistance R_{EQ} is smaller than the smallest resistor.

Passive Sign Convention

Use a positive sign for the power when:

Current is the direction of voltage drop.



Power in a Resistive Element

The power dissipated across two terminals is

$$P = (\pm)VI \quad = (\pm)\frac{V^2}{R} \quad = (\pm)I^2R$$

P = the power in watts
V = the voltage in volts
I = the current in amperes

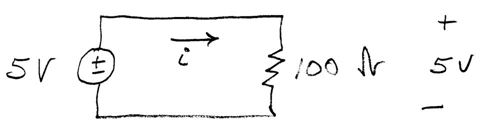
I.A.W. with the Passive Sign Convention

+ (positive) – element is absorbing power.

- (negative) – element is delivering power.

Power - Example

Power delivered/absorbed.



$$P = (+)\frac{V^2}{R} \quad = (+)\frac{(5V)^2}{100\Omega} \quad = (+)\frac{25V}{100\Omega} \quad = (+)\frac{1}{4}Watt$$

Capacitors

Symbol: F for capacitance

Circuit usage: C for capacitor

$$C = \frac{\varepsilon A}{d} \qquad\qquad Q = CV$$

Capacitor resists CHANGE in voltage across it.

Passive charge storage by separation of charge - Electric field energy.

$$i(t) = c\frac{dv}{dt} \qquad\qquad v(t) = \frac{1}{C}\int_{t_0}^{t} i\, d\tau \ + v(t_0)$$

The total energy (in Joules) stored in a capacitor is

$$energy = \frac{CV^2}{2} = \frac{VQ}{2} = \frac{Q^2}{2C}$$

Inductors

Symbol: H for Henries

Circuit usage: L for inductor

$$L = \frac{N\phi}{I}$$

Where N is the number of turns through a magnetic flux ϕ which results from the current I.

Inductor resists CHANGE in current thru it.

Passive energy storage by creation of magnetic field.

$$v(t) = L\frac{di}{dt} \qquad\qquad i(t) = \frac{1}{L}\int_{t_0}^{t} v\, d\tau \ + i(t_0)$$

The total energy (in Joules) stored in an inductor is

$$energy = \frac{LI^2}{2}$$

Capacitors and Inductors in Series and Parallel

Capacitors add in parallel (CAP)

$$C_{EQ} = C_1 + C_2 + C_3 + \ldots$$

Use the following form for series capacitance.

$$C_{EQ} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2} + \ldots}$$

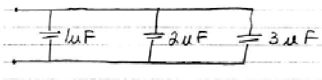
Inductors add in series (just like resistors)

$$L_{EQ} = L_1 + L_2 + L_3 + \ldots$$

Use the following form for parallel inductors

$$L_{EQ} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \ldots}$$

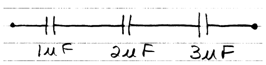
Example - Calculating equivalent capacitance



$C_{eq} = ?$

$$= (1+2+3)\mu F = 6\mu F$$

The equivalent capacitance C_{eq} is larger than the largest capacitor.



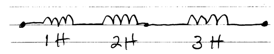
$C_{eq} = ?$

$$= \frac{1}{\frac{1}{1\mu F}+\frac{1}{2\mu F}+\frac{1}{3\mu F}} = \frac{1}{\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}\right)\frac{1}{\mu F}}$$

$$= \frac{1}{\frac{6+3+2}{6}\frac{1}{\mu F}} = \frac{1}{\frac{11}{6}\frac{1}{\mu F}} = \frac{6}{11}\mu F = 0.55\mu F$$

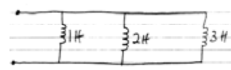
The equivalent capacitance C_{eq} is smaller than the smallest capacitor.

Example - Calculating equivalent inductance



$L_{eq} = ?$ $\quad = (1+2+3)H = 6H$

The equivalent inductance L_{eq} is larger than the largest inductor.



$L_{eq} = ?$

$$= \frac{1}{\frac{1}{1H}+\frac{1}{2H}+\frac{1}{3H}} = \frac{1}{\frac{6+3+2}{6}\frac{1}{H}} = \frac{1}{\frac{11}{6}\frac{1}{H}}$$

$$= \frac{6}{11}H = 0.55H$$

The equivalent inductance L_{eq} is smaller than the smallest inductor.

DC Circuits

Short Break – 5 minutes

DC Circuit Analysis

DC Circuit Analysis include the following topics:

- Ohm's Law
- Kirchhoff's Laws
- Rules for Simple Resistive Circuits
- Superposition Theorem
- Superposition Method
- Loop-Current Method
- Node-Voltage Method
- Source Equivalents
- Maximum Power Transfer

We will also briefly look at RC and RL Transients

Ohm's Law

Ohms Law: V = I R

This version of Ohm's Law assumes a linear circuit.

$$R = \frac{V}{I}$$

$$\Omega = \frac{V}{A}$$

Definitions

Circuit Connections:

Branch – a connection between two elements

Nodes – point of connection of two or more branches.

Loops – any closed path (start/end same point) in a circuit.

Kirchhoff's Laws

Kirchhoff's Current Law – sum of all current equals zero.

sum of all currents in = sum of all currents out.

This is a restatement of conservation of charge.

$$\sum I_{in} = \sum I_{out}$$

Kirchhoff's Voltage Law – sum of all voltages around a closed path is zero.

$$\sum V_{closed\ path} = 0$$

$$\sum V_{rise} = \sum V_{drop}$$

Rules for Resistive Circuits

The current through a simple series circuit is the same in all circuit elements.

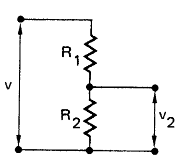
$$I = I_{R_1} = I_{R_2} = I_{R_3}$$

The sum of all voltage drops across all elements is equal to the equivalent applied voltage.

$$V_{EQ} = V_1 + V_2 + \ldots = IR_{EQ}$$

Rules for Simple Resistive Circuits

Voltage Divider for Series Resistors



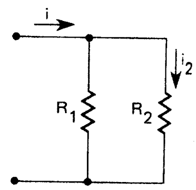
$$v_1 = \frac{R_1}{R_{EQ}} v = \frac{R_1}{R_1 + R_2} v$$

$$v_2 = \frac{R_2}{R_{EQ}} v = \frac{R_2}{R_1 + R_2} v$$

Rules for Simple Resistive Circuits

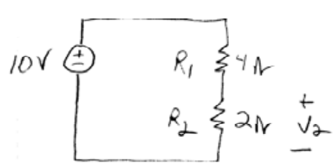
Current Divider for Parallel Resistors:

$$i_1 = \frac{R_2}{R_1 + R_2} i$$



$$i_2 = \frac{R_1}{R_1 + R_2} i$$

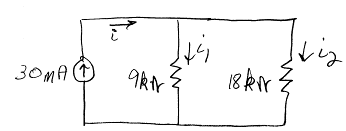
Example - Voltage Divider



Find voltage V_2

$$v_2 = v_s \frac{R_2}{R_1 + R_2} = (10V)\frac{2}{4+2} = (10V)\frac{2}{6} = \frac{10V}{3} = 3.33V$$

Example – Current Divider



Find i_2

$$i_2 = i\frac{R_1}{R_1 + R_2} = (30mA)\frac{9k}{9k + 18k} = (30mA)\frac{9}{27} = \frac{30mA}{3} = 10mA$$

Superposition Theorem

Determine the contribution (responses) of each independent source to the variable in question.

Then sum these responses to find the total response.

The circuit must be linear to use superposition.

Superposition Method

1. Deactivate all independent sources except one.
   - Voltage source = zero when shorted.
   - Current source = zero when opened.
2. Solve the simplified circuit.
3. Repeat until all independent sources are handled.
4. Sum the individual responses to find the total response.

Methods of Analysis

Loop-Current Method – assign currents in a loop and then write the voltages around a closed path (KVL).

Node-Voltage Method – assign a reference voltage point and write current as voltages and resistances at the node. (KCL).

Loop-Current Method

The loop-current is also known as the mesh current method.

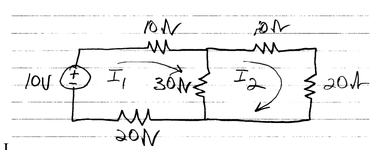
A mesh is a loop which does not contain any other loops within it.

Assign mesh currents to all the (n) meshes.

Apply KVL to each mesh. Express the voltages in terms of Ohm's law – i.e. currents and resistances.

Solve the resulting (n) simultaneous equations.

Example – Loop-current Method



Write the mesh equation for loop I_1.

$$-10V + I_1(10\Omega) + (I_1 - I_2)(30\Omega) + I_1(20\Omega) = 0$$

$$I_1(60\Omega) - I_2(30\Omega) = 10V$$

Write the mesh equation for loop I_2.

$$(I_2 - I_1)(30\Omega) + I_2(10\Omega) + I_2(20\Omega) = 0$$

$$-I_1(30\Omega) + I_2(60\Omega) = 0$$

Solve for the currents.

$$-I_1(30\Omega) + I_2(60\Omega) = 0 \Rightarrow I_2 = \tfrac{1}{2} I_1$$

$$(60\Omega) - \tfrac{1}{2} I_1(30\Omega) = 10V \Rightarrow I_1 = \frac{10V}{45\Omega} = 0.22\,Amps \qquad I_2 = \tfrac{1}{2} I_1 = 0.11\,Amps$$

Node-Voltage Method

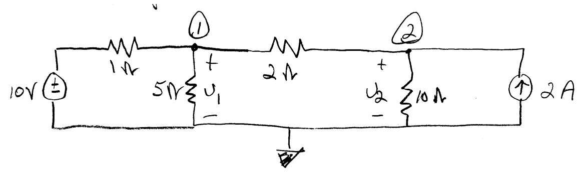
The node-voltage method is also known as nodal analysis.

1. Convert all current sources to voltage sources.
2. Chose a node as the voltage reference node. Usually this is the circuit's signal ground.
3. Write the KCL equations at all unknown nodes

Remember you are writing the sum of all currents entering a node are equal to the sum of all current leaving a node.

A convention is to assume all currents are leaving the node - the direction of the voltage drop is away from the node.

Example – Node-Voltage Method



Write the node equation for at node 1.

$$\frac{V_1-10V}{1\Omega}+\frac{V_1}{5\Omega}++\frac{V_1-V_2}{2\Omega}=0 \qquad V_1\left(\frac{1}{1\Omega}+\frac{1}{5\Omega}+\frac{1}{2\Omega}\right)-\frac{V_2}{2\Omega}=10Amp$$

Write the node equation at node 2.

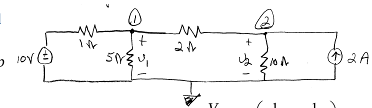
$$\frac{V_2-V_1}{2\Omega}+\frac{V_2}{10\Omega}-2A=0 \qquad -\frac{V_1}{2\Omega}+V_2\left(\frac{1}{2\Omega}+\frac{1}{10\Omega}\right)=2A$$

Solve the simultaneous equations for V_1 and V_2.

Example – Node-Voltage Method

$$V_1\left(\frac{1}{1\Omega}+\frac{1}{5\Omega}+\frac{1}{2\Omega}\right)-\frac{V_2}{2\Omega}=10Amp$$

$$V_1\left(\frac{17}{10\Omega}\right)-\frac{V_2}{2\Omega}=10Amp$$



$$-\frac{V_1}{2\Omega}+V_2\left(\frac{1}{2\Omega}+\frac{1}{10\Omega}\right)=2A$$

$$-\frac{V_1}{2\Omega}+V_2\left(\frac{6}{10\Omega}\right)=2A$$

One way to simplify is the clear the fractions. From node 1, we then have

$$17V_1-5V_2=100V \quad \Rightarrow V_1=\frac{100V+5V_2}{17}$$

Clear the fractions for node 2 equation and Sub above result in the equation for node 2.

$$-5V_1+6V_2=20V \qquad \Rightarrow -5\left(\frac{100V+5V_2}{17}\right)+6V_2=20V$$

$$\Rightarrow\left(\frac{-25}{17}+6\right)V_2=20V+\frac{500}{17}V \quad \Rightarrow 77V_2=340V+500V \quad \Rightarrow V_2=10.91V$$

$$\Rightarrow V_1=9.09V$$

Node-Voltage Method

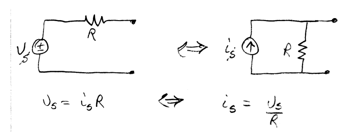
As you see from the last example, even relatively simple equations can require significant time to solve.

Watch your time and tackle time intensive problems only if you have time to spare.

Source Equivalents

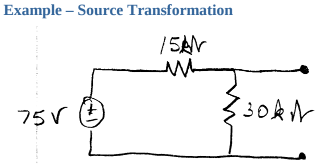
A source transformation (equivalent) exchanges a voltage source with a series resistance with a current source with a parallel resistor.

$$v_s = i_s R$$

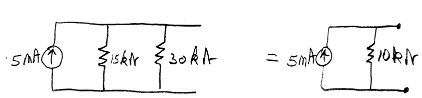


$$i_s = \frac{v_s}{R}$$

Example – Source Transformation

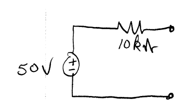


$$i_s = \frac{75\text{ V}}{15k\Omega} = 5\text{ mA}$$



Example – Source Transformation

$$v_s = i_s R = (5mA)(10k\Omega) = 50V$$



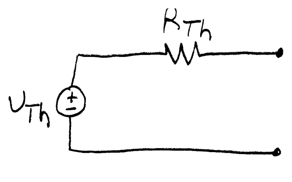
The above equivalent circuit will behave exactly as the original circuit would.

Thevenin's Theorem

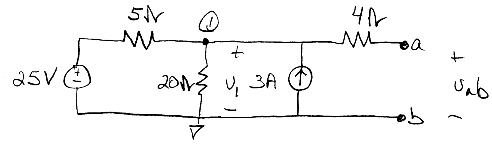
Thevenin's Theorem: a linear two-terminal network can be replace with an equivalent circuit of a single voltage source and a series resistor.

V_{TH} is the open circuit voltage.

R_{TH} is the equivalent resistance of the circuit.



Example – Thevenin's Theorem



Use node analysis to find voltage V_1. Note that $V_1 = V_{TH}$!

$$\frac{V_1 - 25V}{5\Omega} + \frac{V_1}{20\Omega} - 3A = 0$$

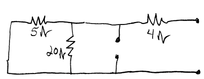
$$4V_1 - 100V + V_1 - 60V = 0$$

$$5V_1 = 160V$$

$$V_1 = V_{TH} = 32V$$

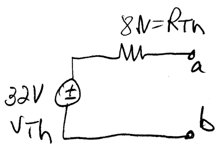
Example – Thevenin's Theorem

Now deactivate all independent sources and find the equivalent resistance.



$5//20 + 4 = 4 + 4 = 8\Omega$

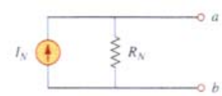
Now we can write the Thevenin equivalent circuit.



Norton's Theorem

Norton's Theorem: a linear two-terminal network can be replaced with an equivalent circuit of a single current source and a parallel resistor.

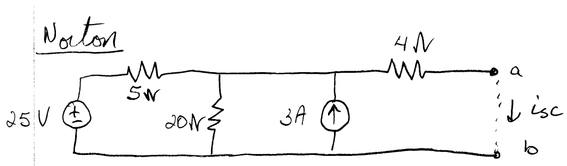
$$I_N = \frac{V_{TH}}{R_{TH}}$$



I_N is the short circuit current.

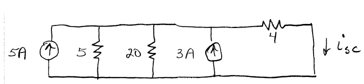
R_{TH} is the equivalent resistance of the circuit.

Example – Norton's Theorem



If you already have the Thevenin equivalent circuit (previous example) - do not start from scratch. However in this example, we will solve the circuit again.

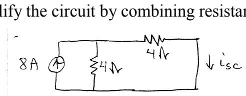
Use a source transformation to put the circuit in terms of current sources.



$$i = \frac{25V}{5\Omega} = 5A \qquad i_{eq} = 5A + 3A = 8A$$

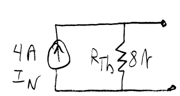
Example – Norton's Theorem

Now simplify the circuit by combining resistances and the current sources



$$5//20 = \frac{5(20)}{5+20}\Omega = 4\Omega$$

The current i_{sc} must be half the 8A input (resistive current divider with equal resistances in each leg.)



Does the previous Thevenin equivalent circuit yield the same answer?

$$I_N = \frac{V_{Th}}{R_{Th}} = \frac{32V}{8\Omega} = \underline{4A}$$

Maximum Power Transfer

The maximum power delivered to a load is when the load resistance equals the Thevenin resistance as seen looking into the source.

$$R_L = R_{TH}$$

The voltage across an arbitrary load is

$$V_L = V_{TH}\frac{R_L}{R_L + R_{TH}}$$

The maximum power delivered to an arbitrary load is given by

$$P_L = \frac{V_L^2}{R_L} = \frac{\left(V_{TH}\frac{R_L}{R_L + R_{TH}}\right)^2}{R_L} = V_{TH}^2\frac{R_L}{\left(R_{TH} + R_L\right)^2}$$

Maximum Power Transfer

The maximum power delivered to a load is when the load resistance equals the Thevenin resistance as seen looking into the source.

$$R_L = R_{TH}$$

When the load resistance equals the Thevenin resistance, the maximum power delivered to the load is given by

$$p_{\max} = \frac{V_{TH}^2}{4R_{TH}}$$

RC and RL Transients

The capacitor and inductor store energy.

A capacitor stores this energy in the form of an electric field.

If a charged capacitor is connected to a resistor, it will give up its energy over a short period of time as follows.

$$v_c(t) = v_c(t=0)e^{-\frac{t}{\tau}}$$

Where τ = RC.

This leads to a short hand rule – a capacitor acts as an open circuit in a DC circuit.

RC and RL Transients

An inductor stores energy in the form of a magnetic field.

If a charged inductor (with a steady current flowing) is connected to a resistor, it will give up its energy over a short period of time as follows.

$$i_L(t) = i_L(t=0)e^{-\frac{t}{\tau}}$$

Where τ = L/R.

This leads to a short hand rule – an inductor acts as a short circuit in a DC circuit.

AC Circuits

AC Circuits include the following topics:

- Alternating Waveforms
- Sine-Cosine Relations
- Phasor Transforms of Sinusoids
- Average Value
- Effective or rms Values
- Phase Angles
- Impedance
- Admittance
- Ohm's Law for AC Circuits
- Complex Power
- Resonance
- Transformers

Alternating Waveforms

The term alternating waveform describes any symmetrical waveform including:

square, sawtooth, triangular, and sinusoidal waves

Sinusoidal waveforms may be given by

$$v(t) = V_{\max}\sin(\omega t + \theta)$$

The phase angle θ describes the value of the sine function at t = 0.

Sine-Cosine Relations

We often need to write sine in terms of cosine and vice versa.

$$\cos(\omega t) = \sin(\omega t + \tfrac{\pi}{2}) = -\sin(\omega t - \tfrac{\pi}{2})$$

$$\sin(\omega t) = \cos(\omega t - \tfrac{\pi}{2}) = -\cos(\omega t + \tfrac{\pi}{2})$$

The period of the waveform is T in seconds.

The angular frequency is ω in radians/second.

The frequency f in hertz is given by

$$f = \frac{1}{T} = \frac{\omega}{2\pi}$$

Phasor Transforms of Sinusoids

A convention is to express the sinusoidal in terms of cosine. Thus the phasor is written as a magnitude and phase under the assumption the underlying sinusoid is a cosine function.

Trigonometric: $V_{\max}\cos(\omega t + \phi)$

Phasor: $V_{eff}\langle\phi$

Rectangular: $V_{real} + jV_{imag} = V_{\max}(\cos\theta + j\sin\theta)$

Exponential: $V_{\max}e^{j\phi}$

Average Value

The average (or mean) value of any periodic variable is given by

$$X_{ave} = \frac{1}{T}\int_0^T x(t)\,dt$$

The average value of a sinusoid is zero.

A waveform may be rectified which results in a different average value.

Effective or rms Value

The root mean squared (rms) of any periodic variable is given by

$$X_{rms} = \sqrt{\frac{1}{T}\int_0^T x^2(t)\,dt}$$

The rms value of a sinusoid is

$$X_{rms} = \frac{X_{\max}}{\sqrt{2}}$$

Phase Angles

It is common to examine the timing of the peak of the volt versus the timing of the peak of the current. This is usually expressed as a *phase shift*.

A capacitor's behavior may be described by its' phasor as $i_c(t) = v_c(t)\angle 90°$.

The current in a capacitor leads the voltage by 90º.

An inductor's behavior may be described by its' phasor as $v_L(t) = i_L(t)\angle 90°$.

The current in an inductor lags the voltage by 90º.

Impedance

The term *impedance* Z in units of ohms describes the effect circuit elements have on magnitude and phase.

$$Z = R \pm jX = Y\langle\theta$$

The resistor has only a real value. Z = R

The capacitor has a negative imaginary value. $Z_c = \frac{1}{j\omega C} = \frac{-j}{\omega C}$

The inductor has a postive imaginary value. $Z_L = j\omega L$

$$R = Z\cos\theta$$

$$X = Z\sin\theta$$

Admittance

The reciprocal of impedance is the complex quantity *admittance* Y.

$$Y = \frac{1}{Z}$$

The reciprocal of resistive part of the impedance is conductance G.

$$G = \frac{1}{R} \qquad B = \frac{1}{X}$$

$$Y = G + jB$$

Ohm's Law for AC Circuits

Ohm's Law for AC circuits is valid when the voltages and currents are expressed in similar manner. Either peak or effective but not a mixture of the two.

V = IZ

Complex Power

The *complex power vector* S is also called the apparent power in units of volts-amps (VA).

S is the vector sum of the real (true, active) power vector P and the imaginary reactive power vector Q.

$$\mathbf{S} = \mathbf{I}^{*}\mathbf{V} = P + jQ$$

The real power P in units of watts (W) is

$$P = \tfrac{1}{2} V_{\max} I_{\max} \cos\theta = V_{rms} I_{rms} \cos\theta = S\cos\theta$$

The reactive power Q in units of volts-amps reactive (VAR) is

$$Q = \tfrac{1}{2} V_{\max} I_{\max} \sin\theta = V_{rms} I_{rms} \sin\theta = S\sin\theta$$

Complex Power

The power factor angle is defined as

$$p.f. = \cos\theta$$

Since the cosine is positive for both positive and negative angles – we must add a description to the power factor angle.

Lagging p.f. is an inductive circuit.

Leading p.f. is a capacitive circuit.

Power factor correction is the proces of adding inductance or capacitance to a circuit in order to achieve a p.f. = cos (0) = 1.

Complex Power

The average power of a purely resistive circuit is

$$P_{ave} = V_{rms} I_{rms} = \frac{V_{rms}^2}{R} = I_{rms}^2 R$$

For a purely reative load p.f. = 0 and the real aveage power = 0.

$$P_{ave} = V_{rms} I_{rms} \cos 90^\circ = 0$$

Resonance

In a resonant circuit at the resonant frequency, the input voltage and current are in phase and the phase angle is zero.

Thus the circuit appear to be purely resistive in its response to this AC voltage (again at the resonant frequency).

The circuit is characterized in terms of resonant frequency ω_0, the bandwidth B, and the quality factor Q.

Each of these parameters can be found from the circuit element values.

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\pi f_0$$

Transformers

Transformers are used to change voltage levels, match impedances, and electrical isolate circuits.

The *turns ratio a* indicates how the magnetic flux links to the mutual inductances of the transformer.

$$a = \frac{N_1}{N_2} = \frac{V_{primary}}{V_{secondary}} = \frac{I_{secondary}}{I_{primary}}$$

A lossless transformer is called an *ideal transformer*. The secondary impedance can be expressed as a reflected impedance given by

$$Z_{EQ} = \frac{V_{primary}}{I_{primary}} = Z_{primary} + a^2 Z_{secondary}$$

Good Luck on the FE/EIT Exam!

It is a time exam. Answer what you know. Mark what you might know and come back later. Do not get bogged down on a few long questions. Move along!

It is a multiple choice exam. Look for hints in the answers.

If totally in doubt – Guess. Use your intuition and science to guess.

FE/EIT Review

Circuits

Instructor: Russ Tatro