

Economics 135
Answers to In-Class Exercises
July 19, 2007

Example 7 Answer:

Suppose you are considering buying a new house and have found a 30-year fixed-rate mortgage with an annual interest rate of 7 percent. What is your monthly payment for the next 30 years on a 400,000 mortgage?

- (1) First convert the annual interest rate to a monthly rate:

$$\begin{aligned}(1+i^m)^{12} &= 1.07 \\ 1+i^m &= (1.07)^{1/12} = 1.005654145 \\ i^m &= 0.005654145\end{aligned}$$

- (2) Next use the formula from lecture and the Chapter 4 Appendix:

$$\begin{aligned}LV &= \frac{FP}{i} \left[1 - \frac{1}{(1+i)^n} \right] \\ \$400,000 &= \frac{FP}{0.005654145} \left[1 - \frac{1}{(1+0.005654145)^{360}} \right] \\ \$400,000 &= \frac{FP}{0.005654145} [0.868632864] \\ \$400,000 &= 153.6276244 FP \\ FP &= \$2,603.698401 = \$2,603.70\end{aligned}$$

Example 8 Answer:

Suppose that you currently owe \$10,000 on a credit card with an annual interest rate of 18 percent ($i = 0.18$). Suppose that you make a constant monthly payment of \$150 and that you never use this credit card again. How many months will it take you to pay off your \$10,000 balance? (Hint: $\ln(x)^n = n \ln(x)$, where \ln is the natural logarithm and x and n are positive numbers).

- (1) First, calculate the monthly interest rate on this credit card:

$$(1+i) = (1+i^m)^{12}$$

$$1.18 = (1+i^m)^{12}$$

$$i^m = (1.18)^{1/12} - 1 = 0.01388843$$

- (2) Apply present value formula for a fixed-payment loan

This credit card is an example of a fixed-payment loan if you pay \$150 every month and never use the credit card again. Using the formula from lecture (and from the Appendix of Chapter 4) yields:

$$LV = \frac{FP}{i} \left[1 - \frac{1}{(1+i)^n} \right]$$

$$\$10,000 = \frac{\$150}{0.01388843} \left[1 - \frac{1}{(1.01388843)^n} \right]$$

$$\$10,000 = \$10,800.35658 \left[1 - \frac{1}{(1.01388843)^n} \right]$$

$$\frac{\$10,000}{\$10,800.35658} = \left[1 - \frac{1}{(1.01388843)^n} \right]$$

$$0.925895356 = \left[1 - \frac{1}{(1.01388843)^n} \right]$$

$$\frac{1}{(1.01388843)^n} = 1 - 0.925895356 = 0.074104644$$

$$1.01388843^n = \frac{1}{0.074104644} = 13.4944303$$

$$n \ln(1.01388843) = \ln(13.4944303)$$

$$n = \frac{\ln(13.4944303)}{\ln(1.01388843)} = \frac{2.602277077}{0.013792869} = 188.6682877$$

Therefore, it will take you almost 189 months to pay off this \$10,000. That's over 15.7 years. If you are 22 years old now, you will be free and clear of this debt by the time you are 38 years old!