Math 108 : Introduction to Formal Mathematics

California State University, Sacramento · Department of Mathematics & Statistics

This course is designed to familiarize students with mathematical deduction on a more rigorous level than that which they encounter in the lower division courses. Very basic facts such as the difference between a conditional ($\Rightarrow$) and a biconditional ($\Leftrightarrow$) are not always clear to them, therefore the course naturally begins with basic logic which is followed by elementary notions of set theory and extended operations on sets. Induction, functions and relations are then covered. The course ends in a 3 or 4 week study of a topic of the instructor's choice.

Catalog Description

Logic of mathematical proof, set theory, relations, functions. Examples and applications from set cardinality, algebra, and analysis. **Graded**: Graded Student. **Units**: 3.0.

Prerequisites

Math 31 and Math 35.

Learning Objectives

The Department of Mathematics & Statistics has a goal in all of its Core Curriculum classes (Math 108, Math 110A/B, and Math 130 A/B) that students be able to understand the vital role that definitions play in the development of formal mathematics. Students must also effectively communicate matheamtical ideas in written form. This could include clear written explanations of matheamtical ideas as well as constructed matheamtical proofs. The writing allows students to reflect upon their learning and deepen their understanding of the concepts in the courses. It is a useful aspect for understanding the language of mathematics and allows students to express themselves clearly in this language.

Math 108 students will be able to:

- Demonstrate an understanding of propositions, including the negation ($\sim P$), conjunction ($P \wedge Q$), disjunction ($P \vee Q$), conditional ($P \Rightarrow Q$), and biconditional ($P \Leftrightarrow Q$) statements, and know vocabulary phrases that are commonly used to translate these connectors.
- Demonstrate an understanding of the converse and contrapositive of the conditional statement, and prove cpnditional statements using the contrapositive.
- Correctly use the universal and existential quantifiers "for every" ($\forall$), "there exists" ($\exists$), and "there exists unique" ($\exists!$), and be able to translate from quantifiers to words and vice versa.
- Negate conjunctions, disjunctions, conditional statements, universal and existential quantified statements, and disprove universally quantified statements with counterexamples.
- Write direct and indirect proofs.
- Demonstrate an understanding of set theory, including set builder notation, the empty set ($\emptyset$), complement ($\overline{A}$), subset ($A \subseteq B$), union ($A \cup B$), intersection ($A \cap B$), set difference ($A - B$), and power set ($\mathcal{P}(A)$). Prove sets are equal. Use indexed family of sets, taking unions, intersections, and their complements, over indexed families of sets.
- Know the equivalence of the Principle of Mathematical Induction, the Principle of Complete Induction, and the Well Ordering Principle, and write proofs using these Principles.

- Demonstrate an understanding of the Cartesian product of sets ($A \times B$), relations between sets, the domain, range, inverse, and composition of relations.
- Demonstrate an understanding of the reflexive, symmetric, and transitive properties of a relation defined on a set. Understand equivalence relations, and be able to prove that a given relation on a set is, or is not, an equivalence relation. Demonstrate a knowledge of equivalence classes, and a partition of a set, and know the connection between equivalence relations and partitions. Understand the equivalence relation of congruence, $\mod n$, on $\mathbb{Z}$, in particular.
- Demonstrate an understanding of functions, functon notation, domain, codomain, range, composition, one-to-one, onto, and inverse functions. Prove functions are, or are not, one-to-one and onto.
- Demonstrate an understanding of, and prove results concerning, the image and inverse image of sets under functions.

Outline

I. Logic (3 Weeks)

Propositions, connectives, truth tables, conditionals, biconditionals, quantifiers, and problems involving all of the above.

II. Set Theory (2 Weeks)

Basic notions of set theory, De Morgan's law, and extended set operations.

III. Induction (1 Week)

Principle of mathematical induction, complete induction, the Well Ordering principle, their equivalence and problem solving.

From this point onward the order in which the topics are covered is up to the instructor.

IV. Functions (3 Weeks)

Functions, range, domain, properties of functions such as one-one and onto, inverse functions, and induced set functions.

V. Relations (3 Weeks)

Cartesian products and relations, functions as relations, equivalence relations and partitions.

The remaining 3 or 4 weeks of the course are spent on putting the acquired techniques to use in any one area where one could prove some nice theorems of relatively short length. Some possible areas are Cardinality, Real Analysis, Geometry, Number Theory, or Algebra.

REMARK: The instructor should be cautioned that certain conditionals, $P \Rightarrow Q$, where the antecedent is false will be very confusing for the students as any such conditional is true. An example is "$x + 1 = 0$ has 2 solutions $\Rightarrow$ 4 is odd" is true. It should be emphasized that this type of conditional is true by virtue of the definition of a conditional and that defining it as such is compatible with the rest of our definitions and axioms. In addition, it would be helpful to explain one of the contexts in which it is often invoked, i.e., to prove $\emptyset \subset A \ \forall A$.