Math 110A : Modern Algebra I

California State University, Sacramento · Department of Mathematics & Statistics

This is the first half of a one year introductory course in Modern Algebra. It is a required course for all students majoring in mathematics. An introduction to groups, rings, fields, and vector spaces will be presented. Some applications may be selected from cryptography, algebraic coding theory, software design, Boolean algebras, electrical circuits, quaternions, finite fields, and constructible numbers, as time permits.

Catalog Description

First half of a one-year introductory course in algebraic concepts. Topics include: groups, subgroups, properties of groups, permutation groups, factor groups, homomorphism theorems. **Graded**: Graded Student. **Units**: 3.0.

Prerequisites

Math 108

Learning Objectives

The Department of Mathematics & Statistics has a goal in all of its Core Curriculum classes (Math 108, Math 110A/B, and Math 130 A/B) that students be able to effectively communicate matheamtical ideas in written form. This could include clear written explanations of matheamtical ideas as well as constructed matheamtical proofs. The writing allows students to reflect upon their learning and deepen their understanding of the concepts in the courses. It is a useful aspect for understanding the language of mathematics and allows students to express themselves clearly in this language.

Math 110A students will be able to:

- Identify whether a specific set along with an operation form a group, and whether a specific subset forms a subgroup.
- Apply basic theorems about orders of elements, groups, and subgroups to specific examples.
- Prove results about orders of elements and generators of cyclic groups.
- Work fluently with examples of permutation groups, using cycle notation, representations of permutations as products of disjoint cycles and as products of transpositions.
- Identify whether a mapping of groups is a homomorphism.
- Use the kernel of a homomorphism to identify whether a homomorphism is a monomorphism.
- Use the correpsondance theorem to identify subgroups and normal subgroups of homomorphic images of groups.
- Work fluently with coset arithmetic in quotient groups.

Course Outline

I. Basic properties of groups (3 weeks)

   A. Binary operations

   B. Identities and inverses

   C. Group tables

   D. Subgroups

   E. Examples chosen from $\mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}, M_{m\times n}(\mathbb{R})$, and $\mathbb{Z}_n$, under addition, and $\mathbb{Q}$, $\mathbb{R}$, $\mathbb{C}$, $GL_n(\mathbb{R})$, $\mathbb{Z}_n^* = \{\overline{a} \in \mathbb{Z}_n \mid (a, n) = 1\}$, $U = \{z \in \mathbb{C} \mid |z| = 1\}$, and $U_n = \{z \in \mathbb{C} \mid z^n = 1\}$, under multiplication

II. Cyclic groups and their subgroups (2 weeks)

   A. Classification of cyclic groups

   B. Subgroups of finite cyclic groups

   C. Subgroup lattice diagrams for $\mathbb{Z}_n$

III. Permutation groups (2 weeks)

   A. Cayleys theorem

   B. Orbits, cycles, and cycle decompositions

   C. Even and odd permutations and A_n

   D. Dihedral groups

IV. Cosets (2 weeks)

   A. Theorem of Lagrange.

   B. The index of a subgroup.

V. Direct Products (1 week)

   A. Fundamental theorem of finite abelian groups (without proof)

VI. Group homomorphisms (4 weeks)

   A. The kernel of a homomorphism.

   B. Normal subgroups.

   C. Factor groups.

   D. Fundamental theorem of group homomorphisms.