Math 130B : Functions of a Real Variable II

California State University, Sacramento · Department of Mathematics & Statistics

This is the second half of a one year upper division course in functions of a real variable that is required of all students majoring in mathematics. The second semester will be devoted to a rigorous development of the theory of Riemann integration, infinite series, and sequences and series of functions.

Catalog Description

Continuation of MATH 130A. This semester will be devoted to a rigorous development of the theory of Riemann integration, infinite series, and sequences and series of functions. **Graded**: Graded Student. **Units**: 3.0.

Prerequisites

Math 130A

Learning Objectives

The Department of Mathematics & Statistics has a goal in all of its Core Curriculum classes (Math 108, Math 110A/B, and Math 130 A/B) that students be able to effectively communicate matheamtical ideas in written form. This could include clear written explanations of matheamtical ideas as well as constructed matheamtical proofs. The writing allows students to reflect upon their learning and deepen their understanding of the concepts in the courses. It is a useful aspect for understanding the language of mathematics and allows students to express themselves clearly in this language.

Math 130B students will be able to:

- Demonstrate an understanding of the fomral $\epsilon - \delta$ definitions of limits, continuity, differentiability ad integrability and be able to establish basic results using these definitions.
- Prove the fundamental theorems, including the Intermediate Value Theorem, Rolle's Theorem, the Mean Value Theorem, l'Hôspital's Rule, and the Fundamental Theorem of Calculus.
- Apply the results of the major theorems to both standard and nonstandard exercises.
- Demonstrate an understanding of the properties of the real numbers, such as finding an infimum and supremum of specific sets, and using the Archimedian property.
- Distinguish between continuity and uniform continuity and understand the connection between differentiability and continuity.
- Identify whether a function is integrable and show that continuous functions on a closed interval are integrable
- Determine whether a sequence satisfies the Cauchy condition and prove whether a sequence converges or diverges.
- Understand the difference between pointwise convergence and uniform convergence for a sequence of real valued functions.

Course Outline

I. Integration (5 Weeks)

   A. Definition of Riemann integral
   B. Linearity and basic properties of the integral
   C. Integrability of continuous and monotone functions
   D. Fundamental theorem of calculus and the indefinite integral
   E. Mean value theorem for integrals
   F. Integration by parts
   G. Riemann integration of piecewise continuous functions
   H. Improper integrals

II. Infinite Series (4 Weeks)

   A. Definition of series
   B. Comparison test and Cauchy convergence
   C. Geometric and alternating series
   D. Absolute and conditional convergence
   E. Ratio, root, and other convergence tests
   F. Multiplication of series

III. Sequences and Series of Functions (5 Weeks)

   A. Pointwise and uniform convergence
   B. Cauchy convergence
   C. Sequences and series of continuous functions
   D. Differentiation, integration and convergence
   E. Uniform convergence tests
   F. Power series
   G. Radius of convergence
   H. Taylor series

IV. Optional Topics

   A. Topology of the real line
   B. Bounded variation and the Riemann-Stieltjes integral